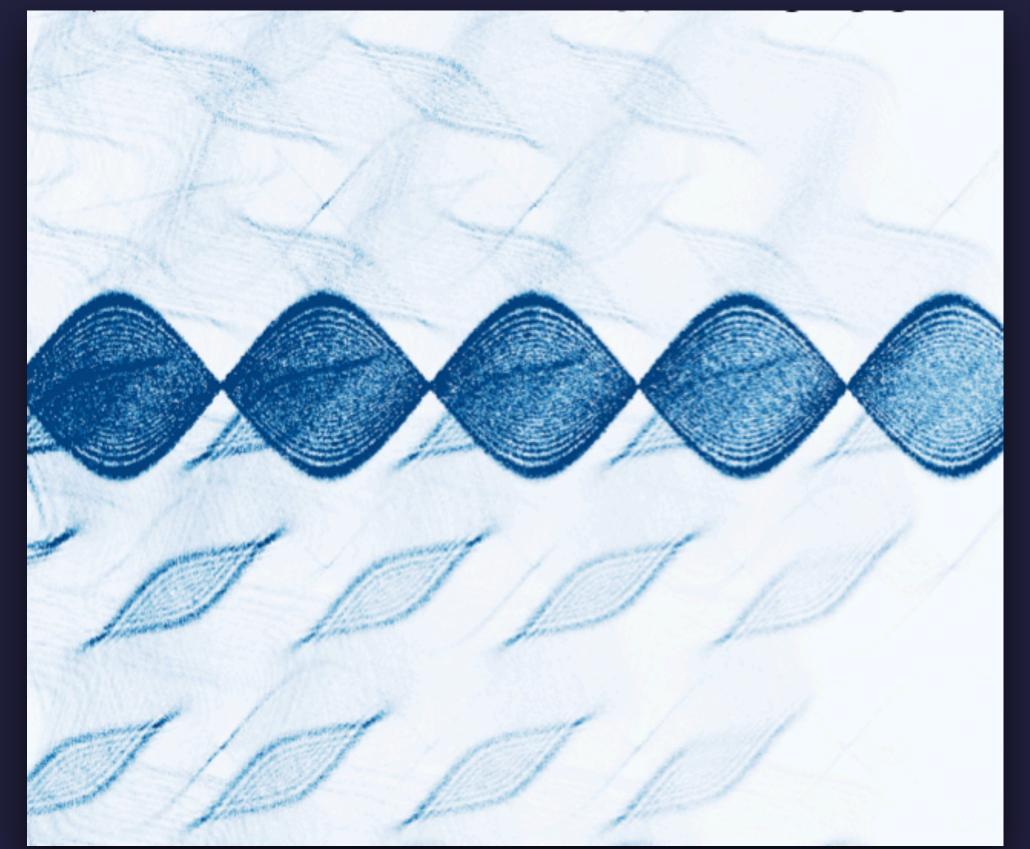
Bloch oscillations in the magnetoconductance of twisted bilayer graphene

with Teresa Vakhtel and Dima Oriekhov

PRB 105, L241408 (2022)





graphene is a playground for textbook quantum effects

- Berry phase
- Klein tunnelling
- atomic collapse
- Zitterbewegung
- Hofstadter butterfly

Bloch oscillations

Über die Quantenmechanik der Elektronen in Kristallgittern.

1929

Von Felix Bloch in Leipzig.

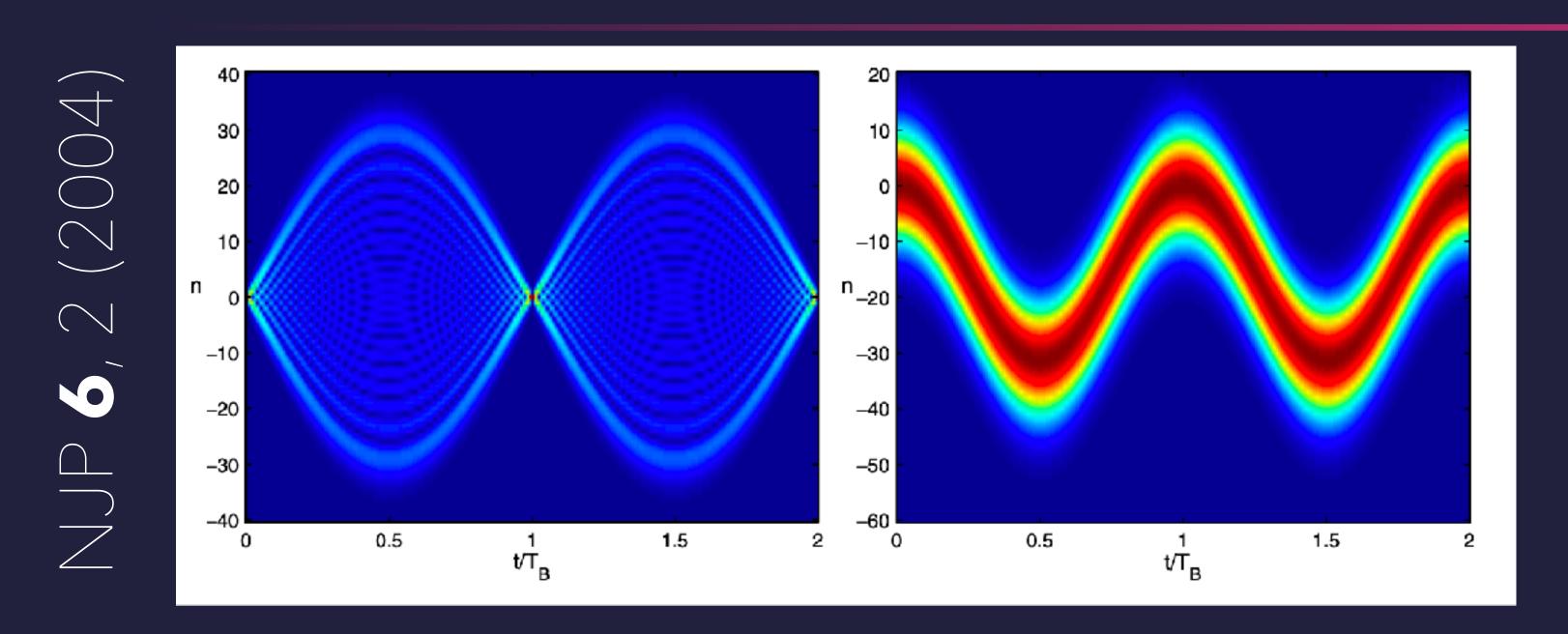
deduced from Bloch's general theory by Zener

A Theory of the Electrical Breakdown of Solid Dielectrics.

1934 CLARENCE ZENER, H. H. Wills Physics Laboratory, Bristol.

Thus if we represent the electron by a wave packet confined to the first energy band, the electron moves in the direction of the field until it is reflected by the lattice, then moves in the opposite direction until it is stopped by the field, whereupon the motion is repeated.

Bloch oscillation of a wave packet



period $T=2\pi \, h/aF$ follows from $2\pi/a$ periodicity of the band structure, and $h \, dk/dt=F$

first observation in 1992 by THz spectroscopy

Optical investigation of Bloch oscillations in a semiconductor superlattice

J. Feldmann, K. Leo, J. Shah, D. A. B. Miller, J. E. Cunningham, T. Meier, G. von Plessen, A. Schulze, P. Thomas, and S. Schmitt-Rink

Phys. Rev. B 46, 7252(R) – Published 15 September 1992

Can we observe it in DC transport?

network model of twisted bilayer graphene (1)

Helical networks in twisted bilayer graphene under interlayer bias

Pablo San-Jose and Elsa Prada Phys. Rev. B **88**, 121408(R) — Published 23 September 2013

Helical network model for twisted bilayer graphene

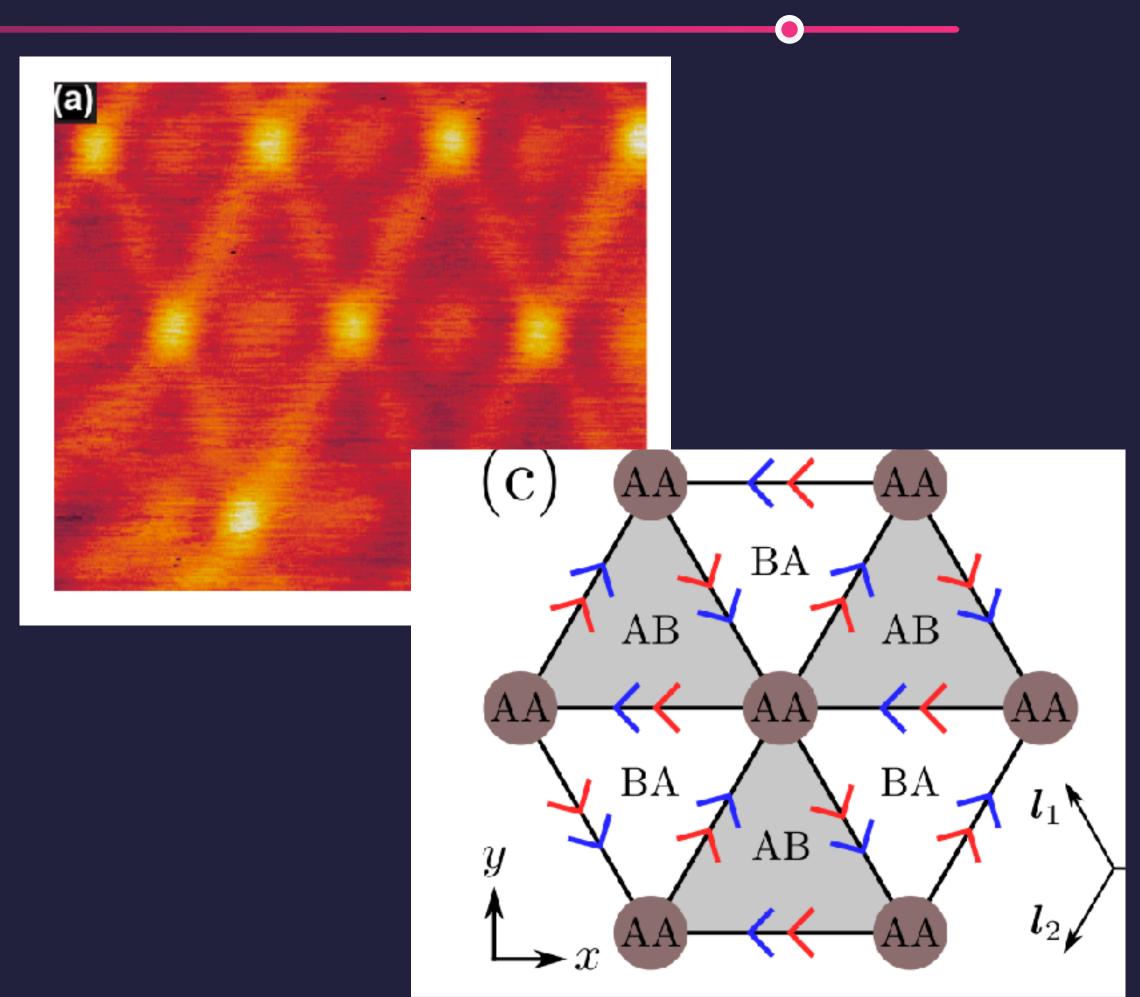
Dmitry K. Efimkin and Allan H. MacDonald Phys. Rev. B **98**, 035404 – Published 2 July 2018

Network model and four-terminal transport in minimally twisted bilayer graphene

Christophe De Beule, Fernando Dominguez, and Patrik Recher Phys. Rev. B **104**, 195410 – Published 8 November 2021

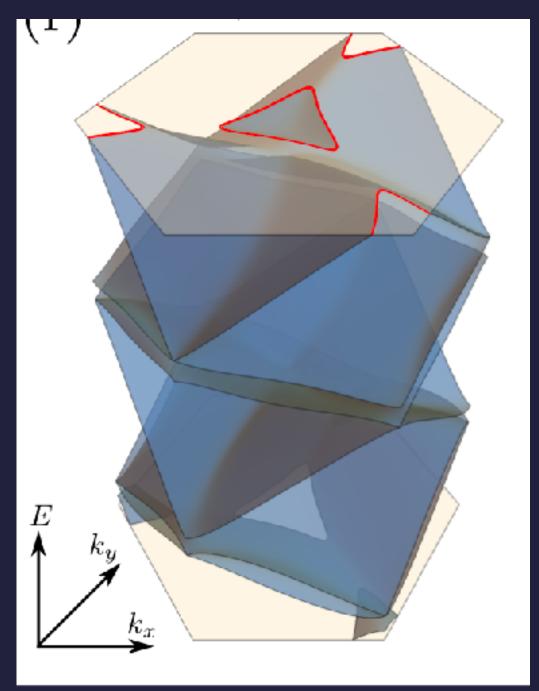
Valley-protected one-dimensional states in small-angle twisted bilayer graphene

J. D. Verbakel, Q. Yao, K. Sotthewes, and H. J. W. Zandvliet Phys. Rev. B **103**, 165134 – Published 23 April 2021

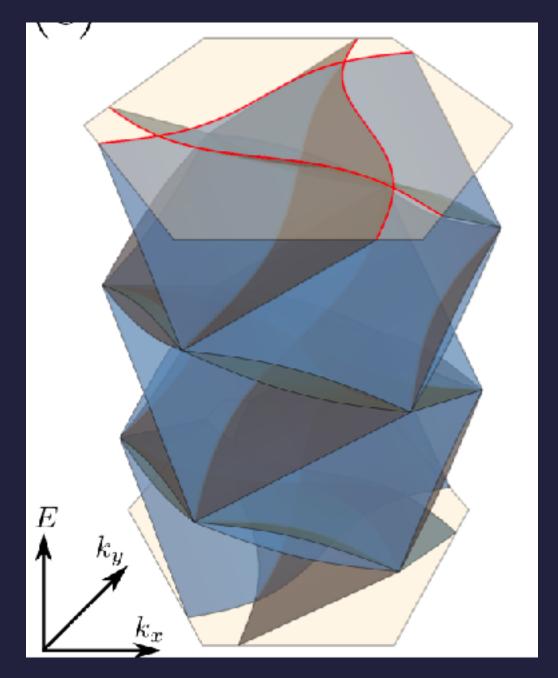


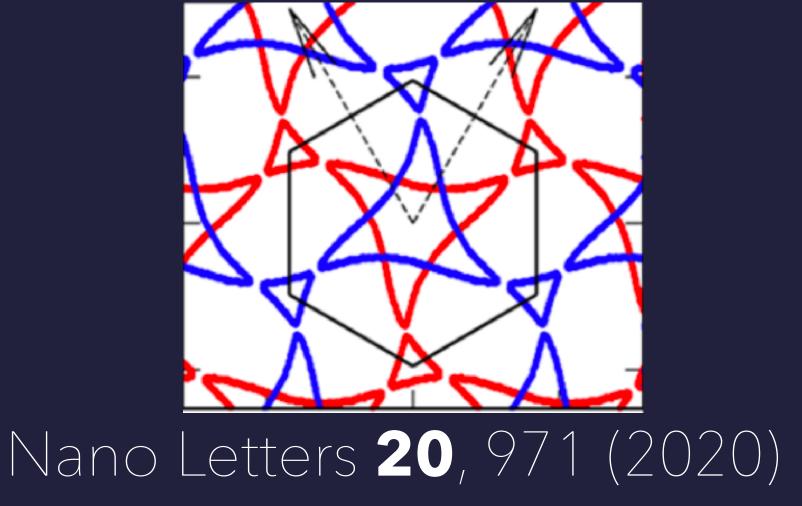
network model of twisted bilayer graphene (2)

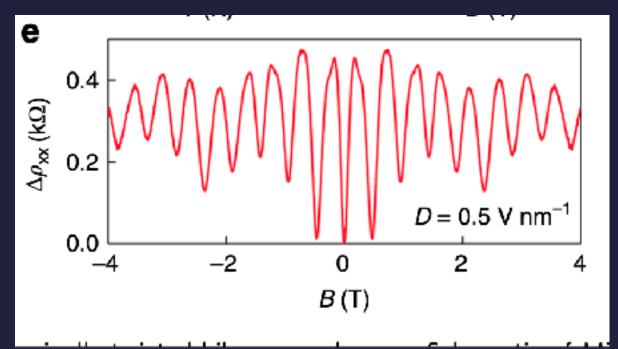
2D regime (closed orbits, no AB osc.)



quasi-1D regime (open orbits, AB osc.)



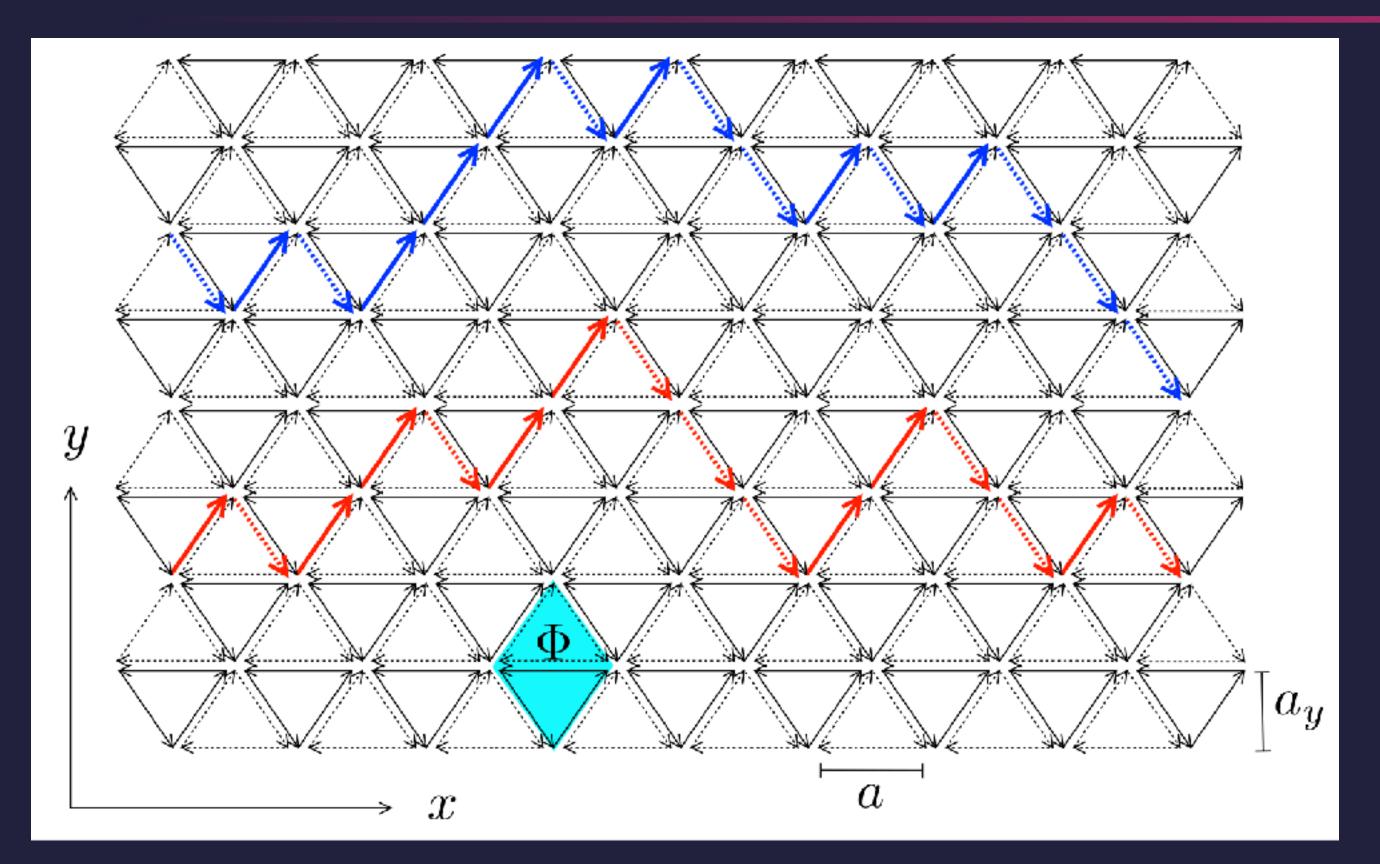




De Beule, Dominguez & Recher, PRL 125, 096402 (2020)

Nature Comm. 10, 4008 (2019)

mapping onto a quantum walk (1)



forward scattering with probability Pf

quasi-1D regime: no loops, x-coordinate increases by a/2 between scattering events, so we can map x onto $t=(2x/a)t_0$

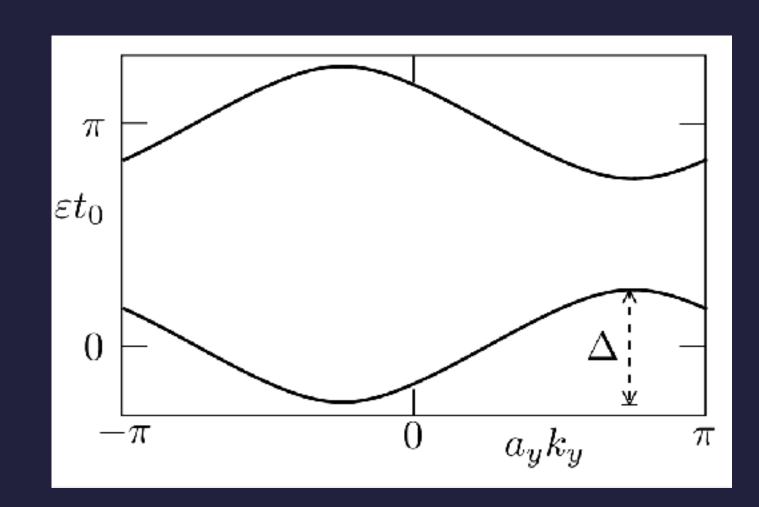
1D quantum walk with time step $t_0=a/v$ and "coin operator"

$$R = \begin{pmatrix} e^{i\pi/4} \sqrt{P_f} & \sqrt{1 - P_f} \\ \sqrt{1 - P_f} & -e^{-i\pi/4} \sqrt{P_f} \end{pmatrix}$$

mapping onto a quantum walk (2)

$$\psi_{t+t_0} = \text{TR}\psi_t, \ \text{T}\psi(y) = e^{-i\alpha_y \hat{k}_y \sigma_z} \psi(y).$$
 eigenvalues $e^{-i\epsilon t_0}$

$$\varepsilon_{\pm} t_0 = \pm \arccos[\sqrt{P_f}\sin(\alpha_y k_y - \pi/4)] + \pi/2$$



include the phase shift from a magnetic field:

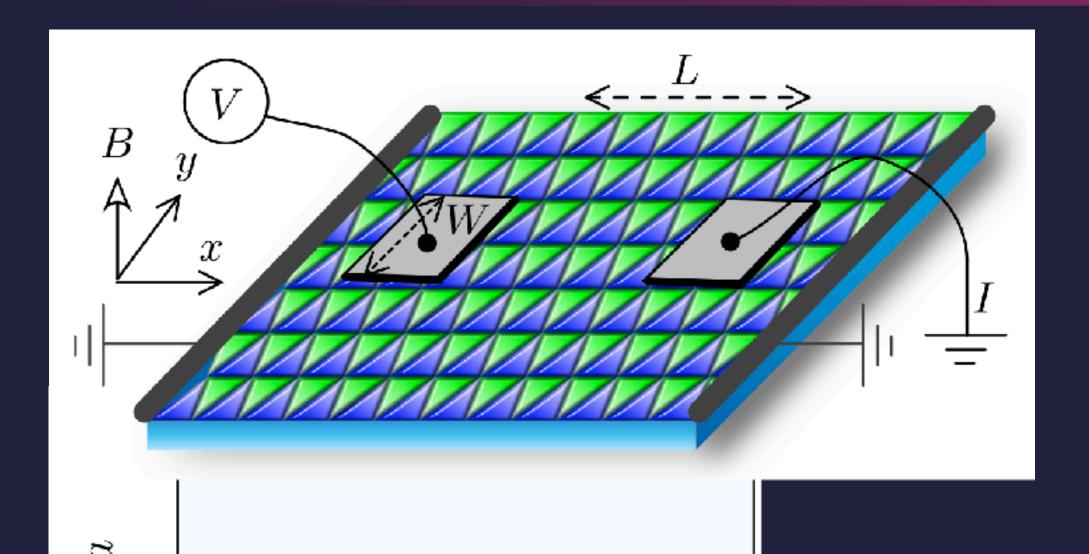
$$\psi_{t+t_0} = e^{i\varphi \hat{y}/\alpha_y} TR\psi_t, \quad \varphi = \pi\Phi/\Phi_0.$$

equivalent to "electric quantum walk"

Quantum walks in weak electric fields and Bloch oscillations

Pablo Arnault, Benjamin Pepper, and A. Pérez Phys. Rev. A **101**, 062324 – Published 19 June 2020

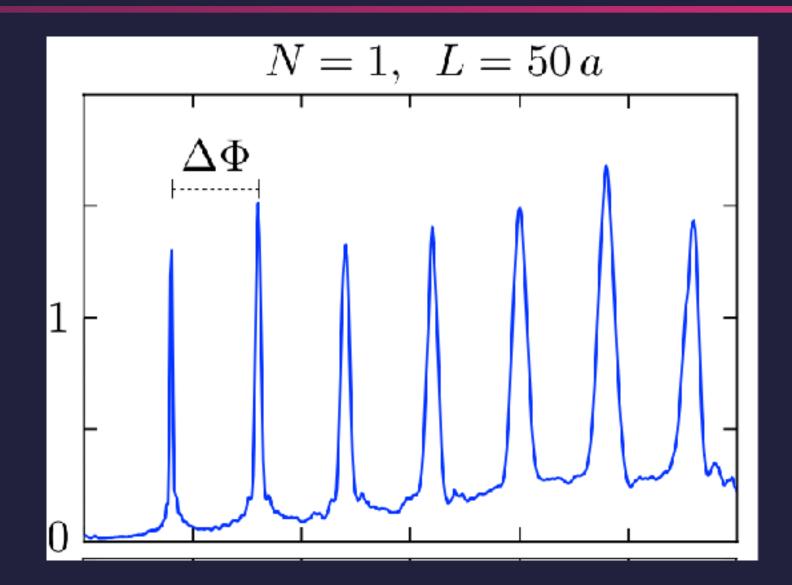
Bloch oscillations in DC transport



x/a 400

200

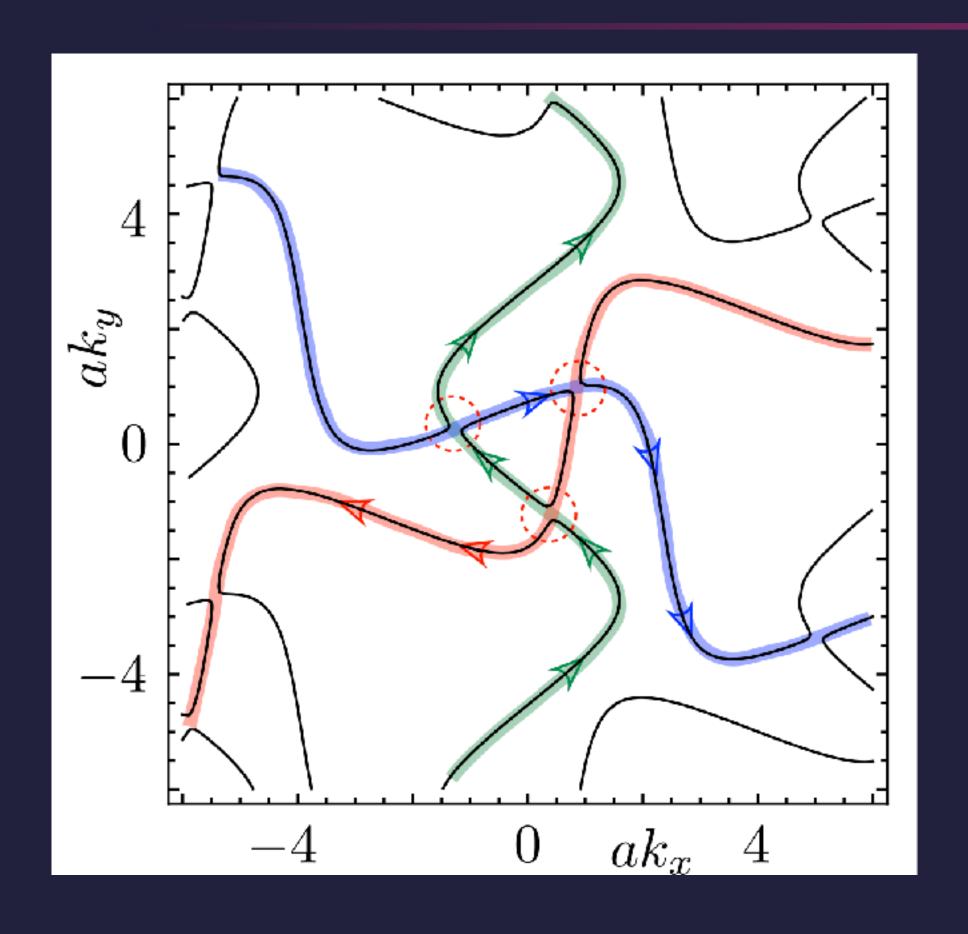
-200



weak-field magnetoconductance oscillations with narrow contacts ($W \approx 0.25 \mu m$, $L \approx 7 \mu m$)

$$\Delta B = (h/e)(a_y L)^{-1} \approx 2.4 \,\text{mT}$$

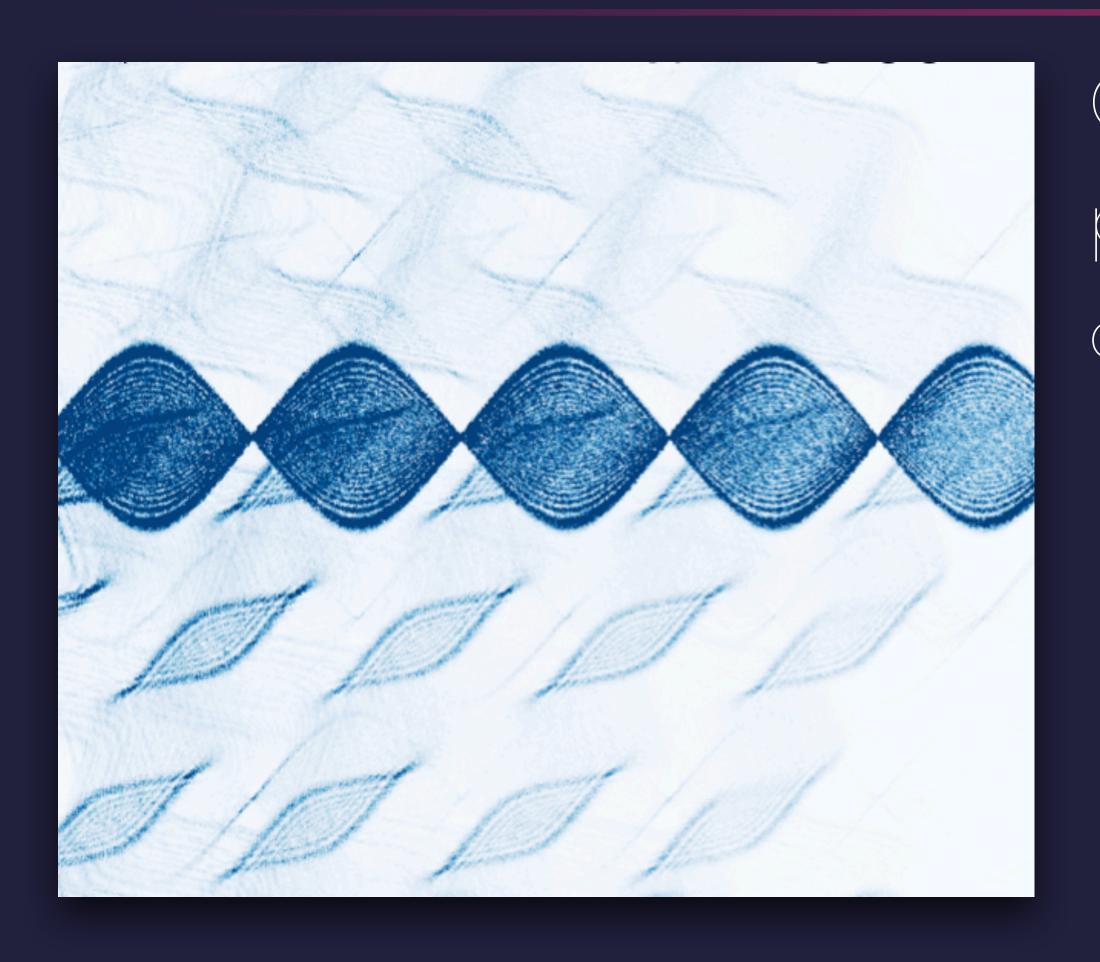
Magnetic breakdown



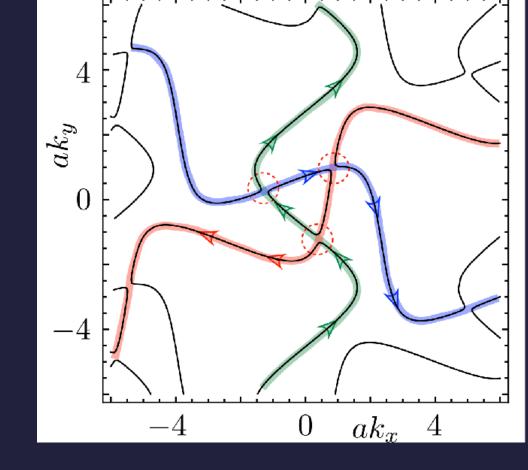
A magnetic field drives a wave packet in the direction of the arrows. Points of magnetic breakdown (tunneling between two equienergy contours) are encircled. The resulting open orbits are responsible for the Bloch oscillations.

1 ()

Magnetic breakdown



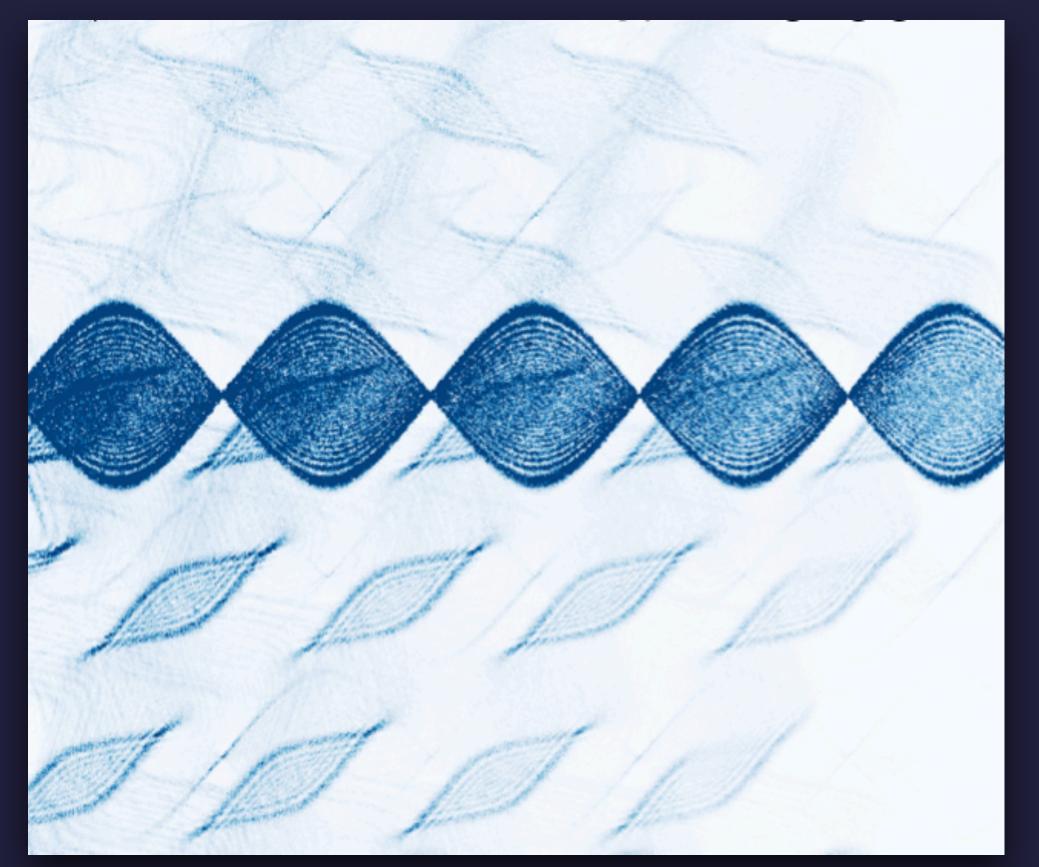
Coupling of different open orbits produces "side branches" of the Bloch oscillations.

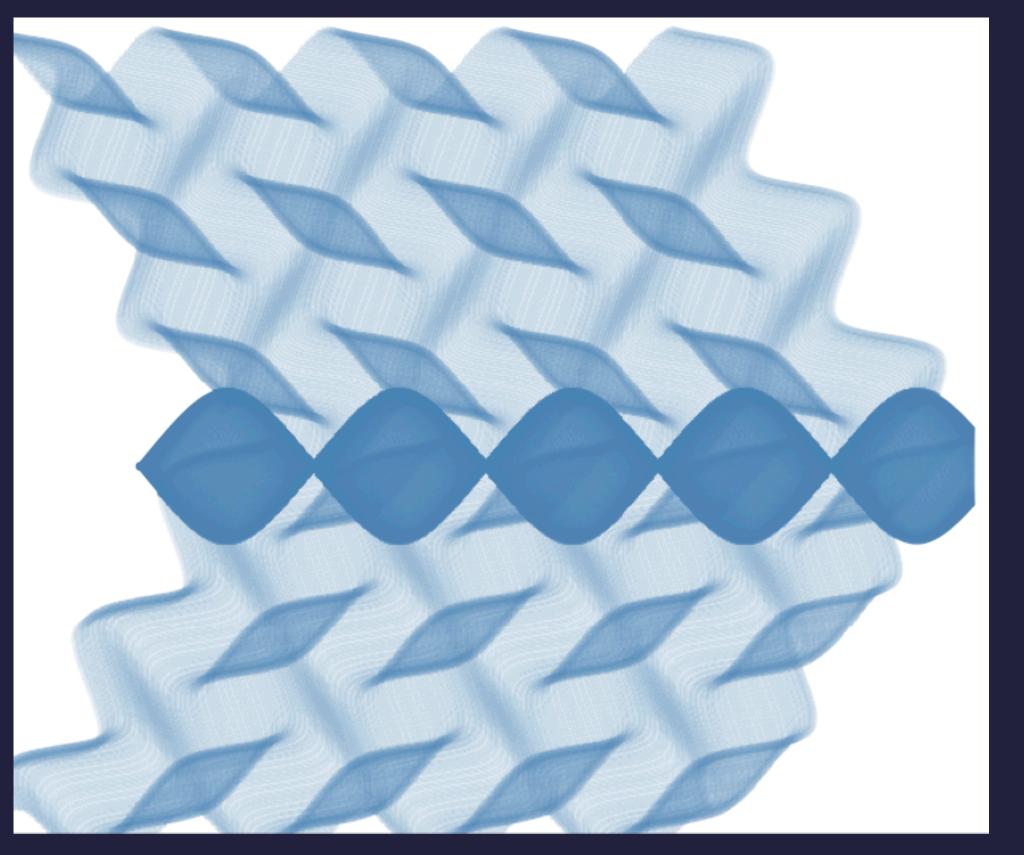


what is this "beating" effect? quantum interference???

11

Magnetic breakdown





semiclassical theory: PRB 106, 235413 (2022)

Bloch oscillations go magnetic

- Bloch oscillations can be observed in DC transport in the network of domain walls in twisted bilayer graphene
- mapping of open orbits in a magnetic field onto electric quantum walk
- the magnetic field confines the electrons sideways over a length $\propto 1/B$
- breathing mode observable as weak-field (mT) magnetoconductance oscillations
- point contact needed to resolve the oscillations