PCS Center for Theoretical Physics of Complex Systems

ibs Institute for Basic Science

flat band physics

Sergej Flach
Center for Theoretical Physics of Complex Systems
Institute for Basic Science
Daejeon South Korea

1. single particle flat band physics
2. many body flat band physics

Adv Phys: X 3 1473052 (2018) ; APL PHOTONICS 3 070901 (2018)

flatbands @ PCS

Yeongjun Kim

Sanghoon Lee

Tilen Cadez

Alexei Andreanov

Chang Hwan Yi

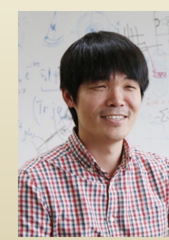
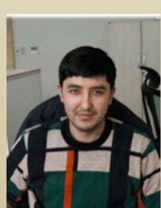
Jung Wan Ryu

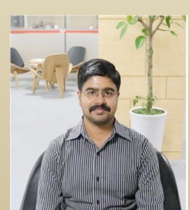


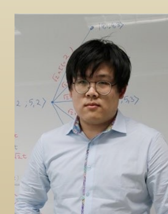




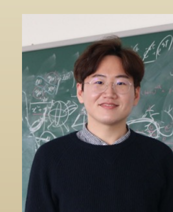










flat band physics

1. single particle flat band physics

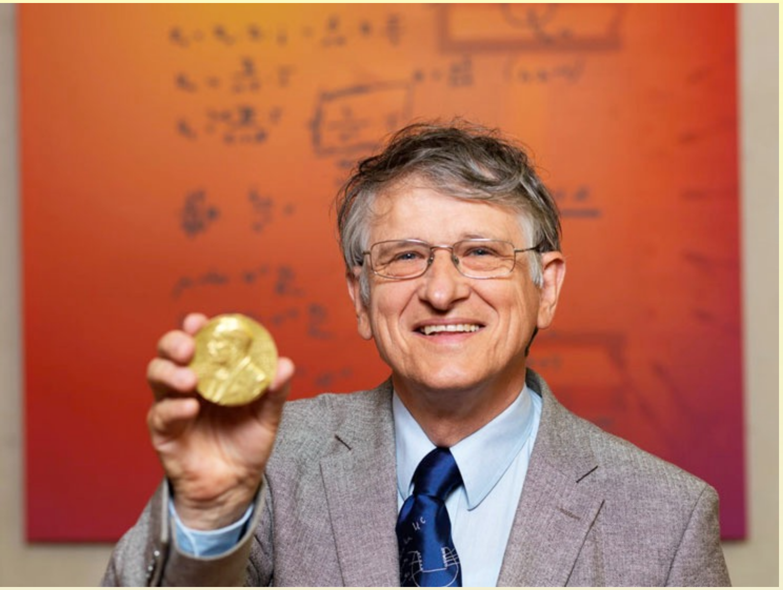
Sergej Flach
Center for Theoretical Physics of Complex Systems
Institute for Basic Science
Daejeon South Korea

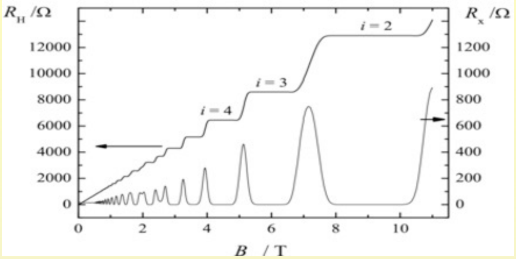
1. flat bands (FB): basics
2. compact localized eigenstates (CLS)
3. All Bands Flat (ABF), chiral FBs, anti-PT FBs
4. Wannier-Stark FBs
5. disorder
6. experiments

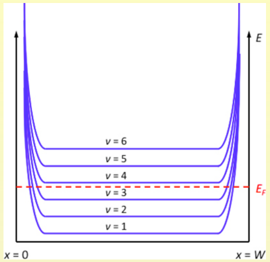
Adv Phys: X 3 1473052 (2018) ; APL PHOTONICS 3 070901 (2018)

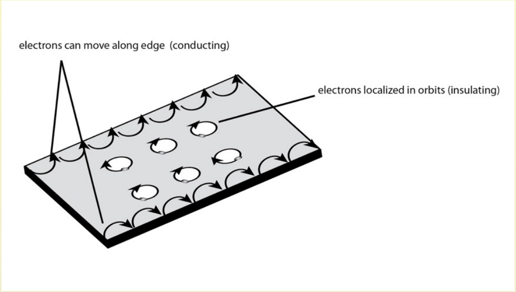
What is the most famous **flat band ???**

I will not talk on this:







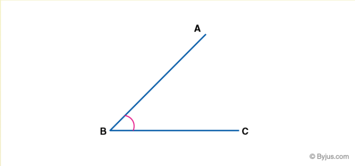


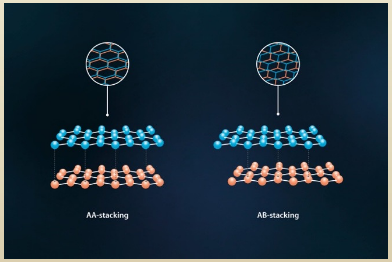
Klaus von Klitzing 1985
2d electron gas in magnetic field
Quantum Hall effect
Quantized conductance
Landau levels
Protected by topology

ALL BANDS FLAT !!!

But this is NOT what I will discuss.
Though we might get back to it

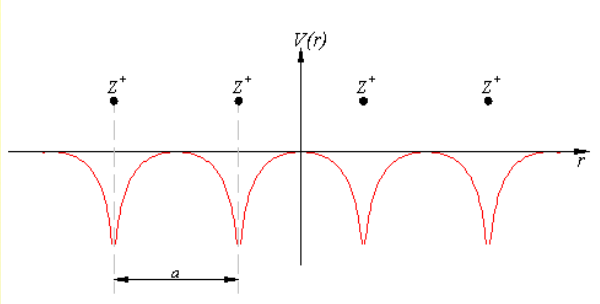
And I will not talk on that:





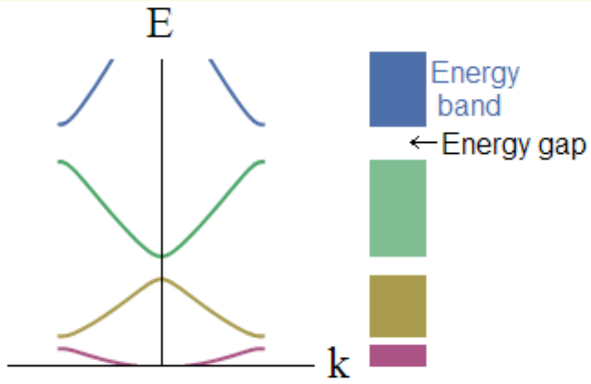
Waves in periodic structures

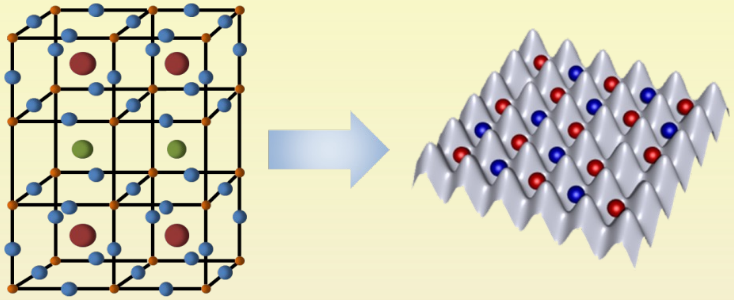
$$i\partial_t \psi(\mathbf{r},t) + \nabla^2 \psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) = 0,$$

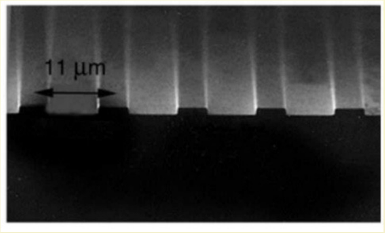


http:/en.wikipedia.org

Band structure

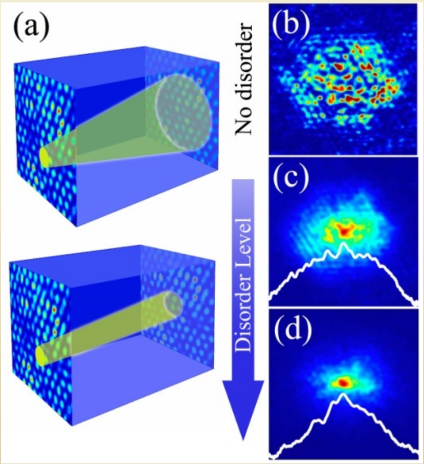






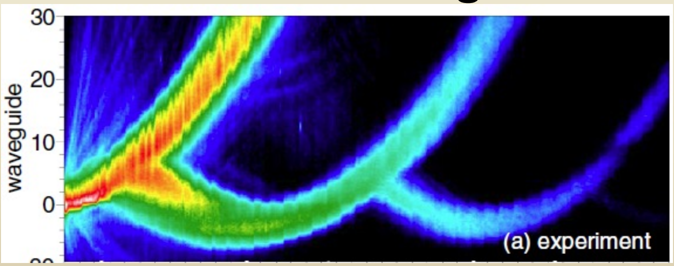
http://darpa.mil

Anderson localization



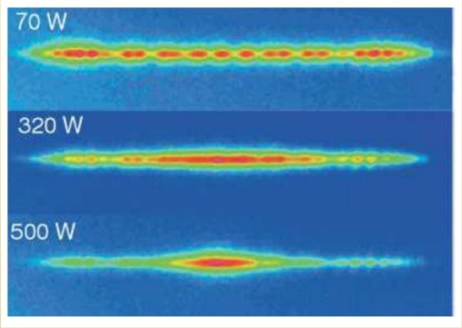
T. Schwartz et al., *Nature* 446, 52 (2007)

Bloch oscillations & Landau-Zener tunnelling



H. Trompeter et al., PRL 96, 023901 (2006); PRL 96, 053903 (2006)

Discrete solitons



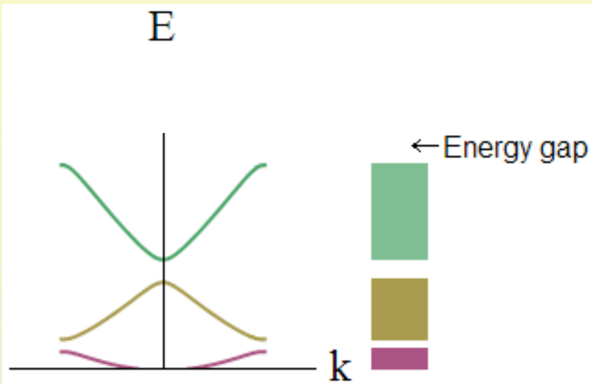
D. N. Christodoulides et al., Nature **424** 817 (2003)

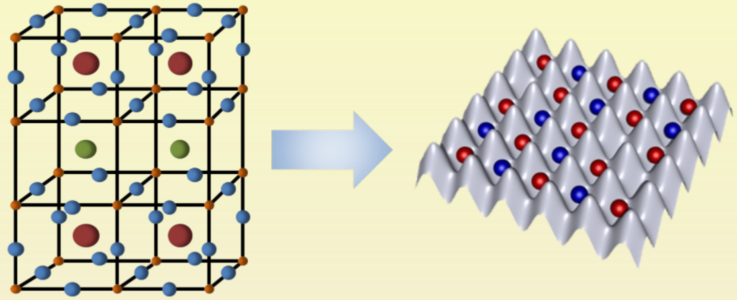
Waves on lattices

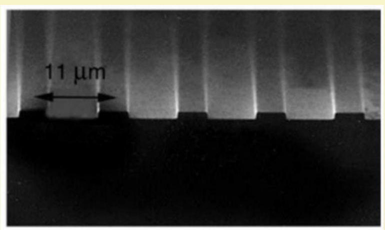
$$i\dot{\psi}_{l,\mu} + \sum_{m,\mu'} t_{m,l,\mu,\mu'}\psi_{m,\mu'} = 0$$

$$E\phi_{l,\mu} = -\sum_{m,\mu'} t_{m,l,\mu,\mu'}\phi_{m,\mu'}$$

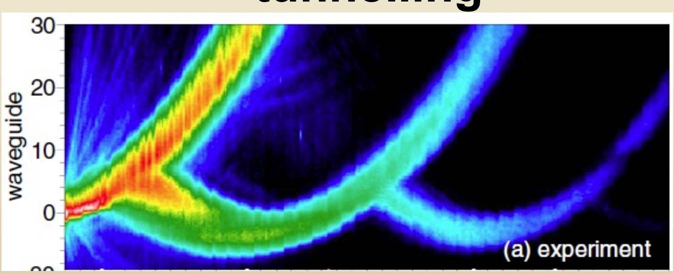
Band structure





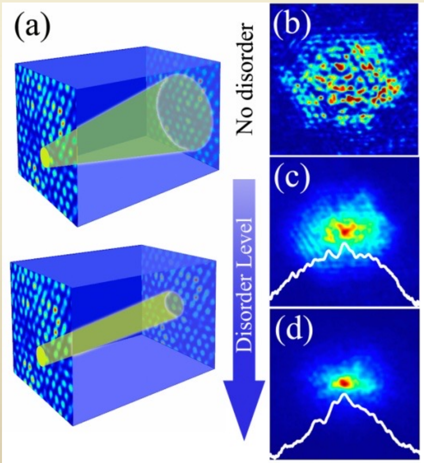


Bloch oscillations & Landau-Zener tunnelling



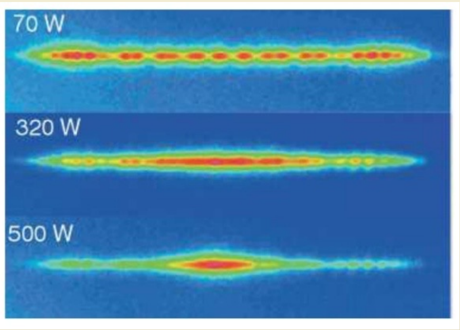
H. Trompeter et al., PRL 96, 023901 (2006); PRL 96, 053903 (2006)

Anderson localization



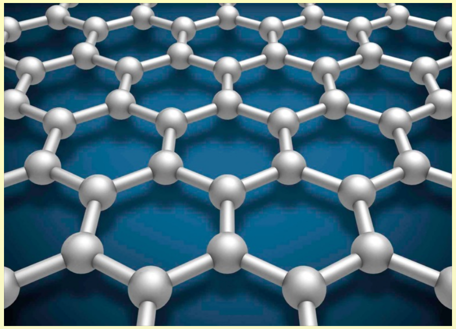
T. Schwartz et al., *Nature* 446, 52 (2007)

Discrete solitons

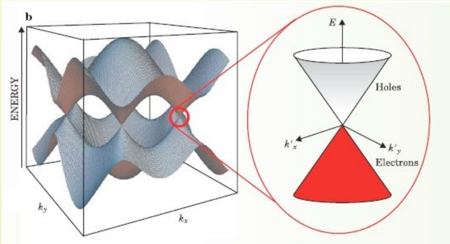


D. N. Christodoulides et al., Nature **424** 817 (2003)

Tight Binding Networks:



$$i\dot{\psi}_l = \boxed{} - \sum_N t_{l,l'}\psi_{l'}$$



$$\psi_l(t) = \phi_l \mathrm{e}^{-iEt}$$

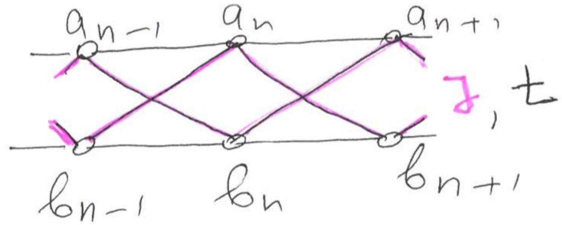
$$E\phi_l = \boxed{} - \sum_N t_{l,l'}\phi_{l'}$$

Consider periodic lattice with discrete translational invariance:

$$\text{band structure } E_\mu(k)$$

$$1 \le \mu \le \nu$$

$$\text{flat band } : \nabla_k E_{\mu_0}(k) = 0 \text{ for any } k$$



$$\begin{cases} E\,a_n = -a_{n+1} - a_{n-1} - \gimel\, b_{n+1} - \gimel\, b_{n-1} \\ E\,b_n = -b_{n+1} - b_{n-1} - \gimel\, a_{n+1} - \gimel\, a_{n-1} \end{cases}$$

Floquet, Bloch: $\begin{Bmatrix} a_n \\ b_n \end{Bmatrix}_k = e^{ikn} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix}$

$\Downarrow$

$$\begin{cases} E_k\,\alpha = -e^{ik}\alpha - e^{-ik}\alpha - \gimel\, e^{ik}\beta - \gimel\, e^{-ik}\beta \\ E_k\,\beta = -e^{ik}\beta - e^{-ik}\beta - \gimel\, e^{ik}\alpha - \gimel\, e^{-ik}\alpha \end{cases}$$

$$E_k \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = H_k \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad H_k = -2\cos k \begin{pmatrix} 1 & \gimel \\ \gimel & 1 \end{pmatrix}$$

$$\boxed{E_\pm(k) = 2\cos k \cdot (-1 \pm \gimel)}$$

$J = 1$: $E_+(k) = E_{FB} = 0$!

true for all k

↳ macroscopic degeneracy

↳ $\sum_k c_k \begin{Bmatrix} a_n \\ b_n \end{Bmatrix}_k$ also FB eigenvector!

$$\Downarrow \quad \begin{Bmatrix} a_n \\ b_n \end{Bmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \delta_{n,0}$$

Compact Localized (Eigen) State

CLS

Invert the procedure!

If we find one CLS, we find a macroscopic number!

All to the same eigenvalue!

So we know then there must be a Flat Band!

Without the painful transformation into k-space!

Without the need to diagonalize matrices!

How can we find the CLS?

Flach,Leykam,Bodyfelt,Matthies,Desyatnikov, EPL (2014)

Detangling U=1

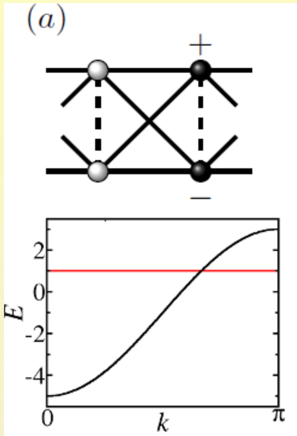
$$E\,a_n = \quad a_n - a_{n+1} - a_{n-1} - b_{n-1} - b_{n+1} - t\,b_n,$$
$$E\,b_n = \quad b_n - a_{n+1} - a_{n-1} - b_{n-1} - b_{n+1} - t\,a_n.$$

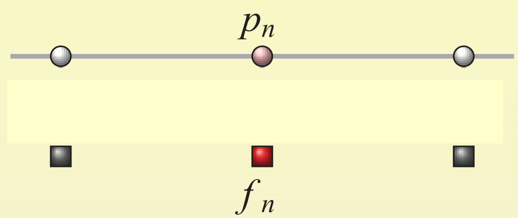
$$p_n = \frac{1}{\sqrt{2}}(a_n + b_n)\,, \qquad f_n = \frac{1}{\sqrt{2}}(a_n - b_n),$$

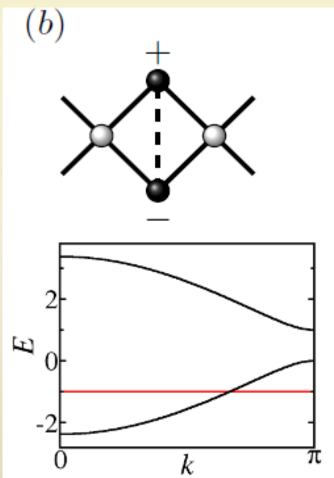
$$E\,p_n = (\quad - t)\,p_n \qquad - 2\,(p_{n+1} + p_{n-1})$$
$$E\,f_n = (\quad + t)\,f_n$$

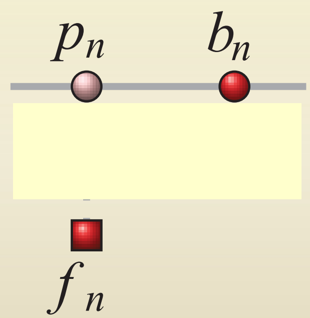
Flach,Leykam,Bodyfelt,Matthies,Desyatnikov, EPL (2014)

Detangling U=1







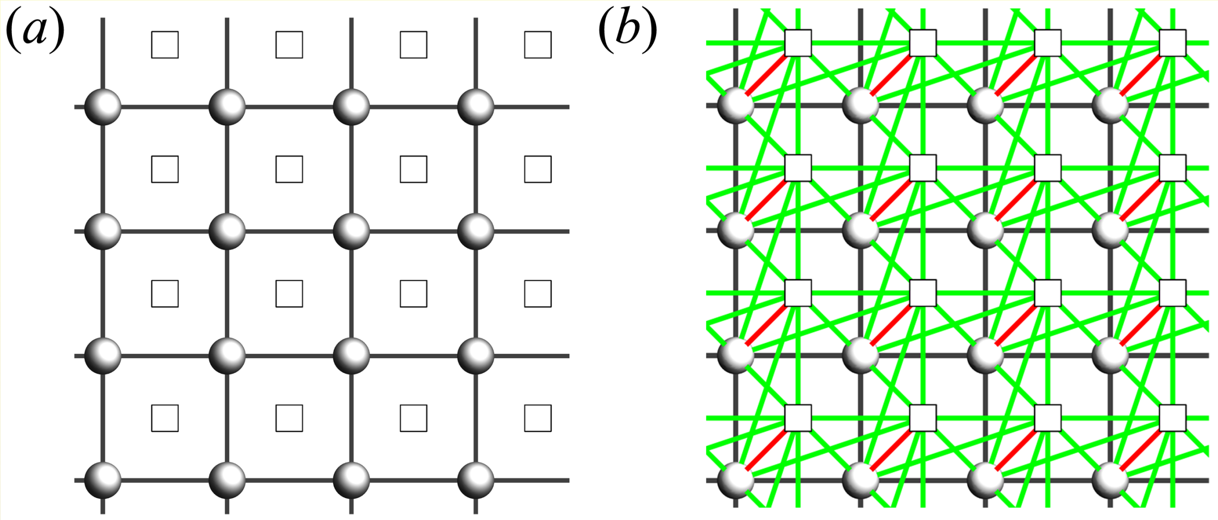


Apply unitary transformation to each unit cell:

$$\tilde{\vec{\psi}}_n = \mathcal{U}\vec{\psi}_n$$

Flach,Leykam,Bodyfelt,Matthies,Desyatnikov, EPL (2014)

Invert the procedure, generalize to all U=1 classes of models !



This is the most general Flat Band Generator in any lattice dimension d and any hopping range for class U=1

$$\vec{\psi}_n = \mathcal{U}^{-1}\tilde{\vec{\psi}}_n$$

Some formula for d=1

$\vec{\psi}_n$ is a vector with components $\psi_{\mu,n}$

$\mu = 1, 2, ..., \nu$

$E\vec{\psi}_n = \sum_m H_{n-m}\vec{\psi}_{n-m}$, $H_m = H^{\dagger}_{-m}$

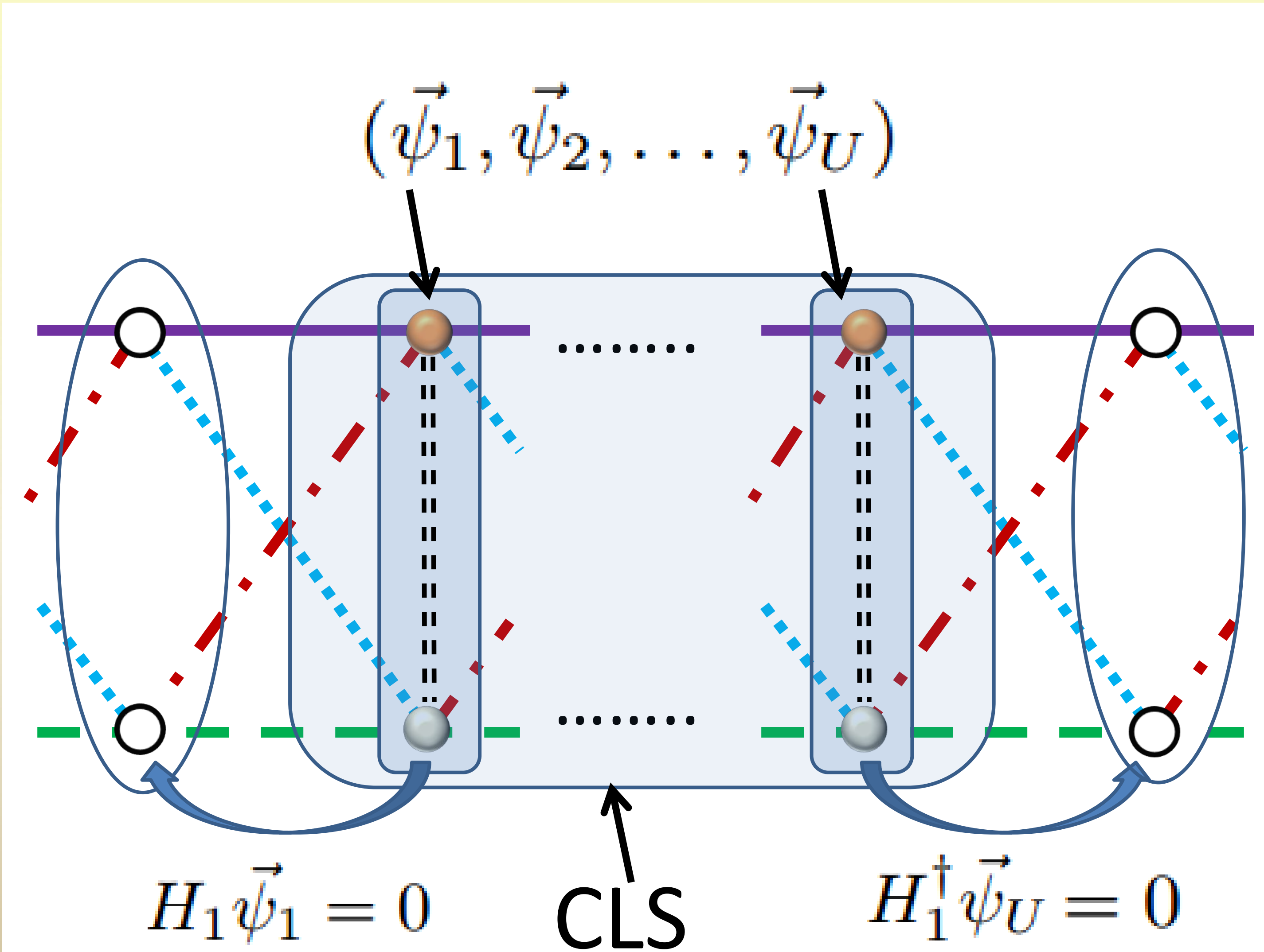
$H_m \equiv 0$ for $|m| > |m_c$

$m_c = 1$:

$$H_0\vec{\psi}_n + H_1\vec{\psi}_{n+1} + H_1^{\dagger}\vec{\psi}_{n-1} = E\vec{\psi}_n$$

Maimaiti,Andreanov,Park,Gendelman,SF, PRB 2017

$$\nu = 2\ ,\ m_c = 1$$



Maimaiti,Andreanov,Park,Gendelman,SF, PRB 2017

The most general FB Tester in one dimension

Sufficient condition for the existence of a CLS with $m_c = 1$: tridiagonal $U \times U$ block matrix

$$\begin{pmatrix} H_0 & H_1 & 0 & 0 & \dots & 0 \\ H_1^\dagger & H_0 & H_1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & & \vdots \\ 0 & \dots & 0 & H_1^\dagger & H_0 & H_1 \\ 0 & \dots & 0 & 0 & H_1^\dagger & H_0 \end{pmatrix}$$

possesses CLS as eigenvector $(\vec{\psi}_1, \vec{\psi}_2, \dots, \vec{\psi}_U)$ and

$$H_1 \vec{\psi}_U = H_1^\dagger \vec{\psi}_1 = 0 \ .$$

Maimaiti,Andreanov,Park,Gendelman,SF, PRB 2017

Turning the FB Tester into a systematic local new FB Generator for d=1

$U = 1,\ \nu = 2,\ m_c = 1$:

$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},\ H_1 = \begin{pmatrix} 0 & 0 \\ 0 & |d|e^{i\varphi} \end{pmatrix}$$

We obtain all known FB models (e.g. cross-stitch) plus their time-reversal symmetry broken extensions!

U=2, two bands, d=1

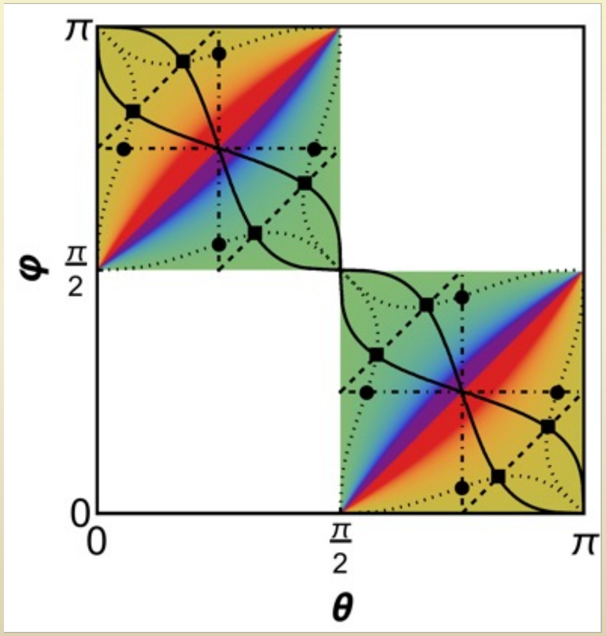
$$U = 2,\ \nu = 2,\ m_c = 1:$$

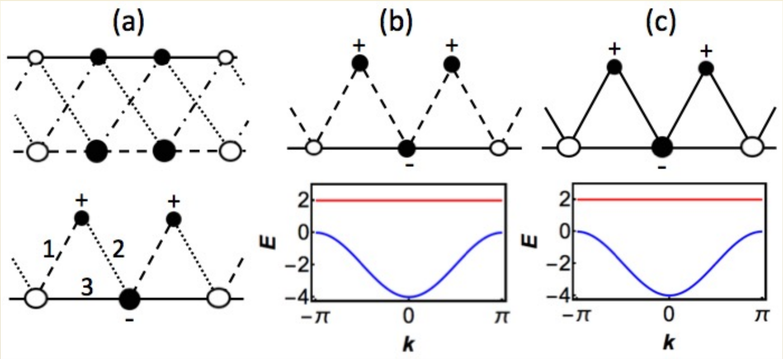
$$H_1 = \alpha|\theta,\delta\rangle\langle\varphi,\gamma|$$

$$|\varphi,\gamma\rangle = \begin{pmatrix} \cos\varphi \\ e^{i\gamma}\sin\varphi \end{pmatrix}$$

$$|\theta,\delta\rangle = \begin{pmatrix} \cos\theta \\ e^{i\delta}\sin\theta \end{pmatrix}$$

Flat band energy





Basis rotation yields generalized sawtooth
v/h lines: zero onsite energies
Curved lines: two hoppings equal

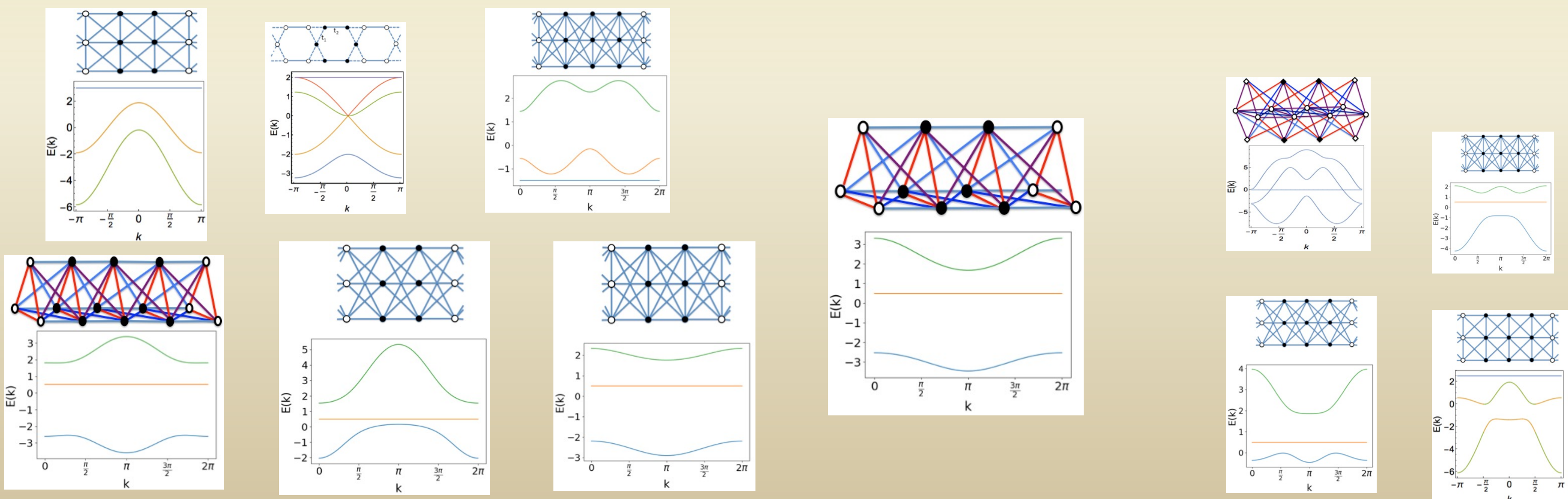
(a) Generalized sawtooth (ST)
(b) known ST : filled circles
(c) new ST : filled squares

Generalizing FB Generator for d=1 to any number of bands and any U

PHYSICAL REVIEW B **99**, 125129 (2019)

Universal $d = 1$ flat band generator from compact localized states

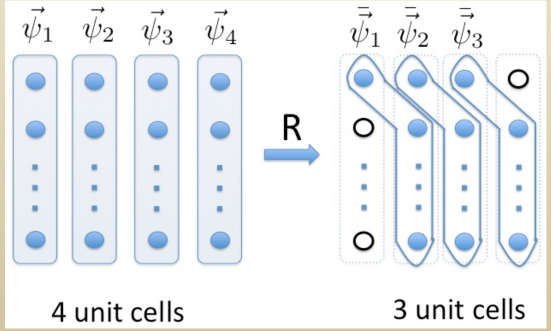
Wulayimu Maimaiti,[1,2] Sergej Flach,[1] and Alexei Andreanov[1]

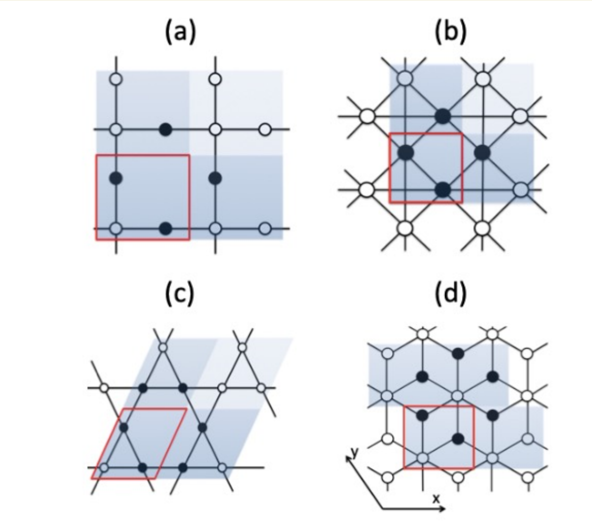


Extending FB Generator to d=2, and more: beautiful results!

Wulayimu Maimaiti, Alexei Andreanov, SF, PRB 103, 165116 (2021)

- **Shape of CLS relevant classifier in d=2**
- **All Bands Flat: U=1 in suitable basis!**





flat bands: a needle in the haystack with a surprising internal structure

- you do not simply run into a FB lattice Hamiltonian
- A typical lattice Hamiltonian will lack FBs
- finetuning in H-space of lattice Hamiltonians for FBs
- symmetries in H-space: predict and protect FBs
- perturb FB Hamiltonians off finetuned subspaces
- expect many exotic phases with phase transition hypersurfaces
- transition surfaces will originate from FB H-subspace
- FB H-subspaces act like singularities
- finetune perturbations or many body interactions !

Flat Bands and Compact Localized States

Finite hopping range, finite band number

A Flat Band

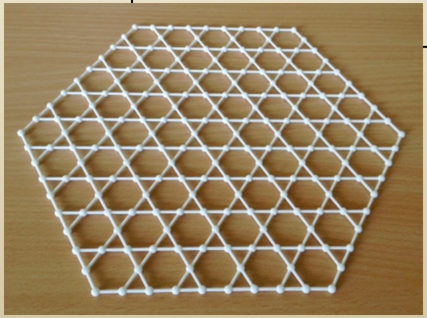
COMPACT LOCALIZED STATES as FB eigenstates!

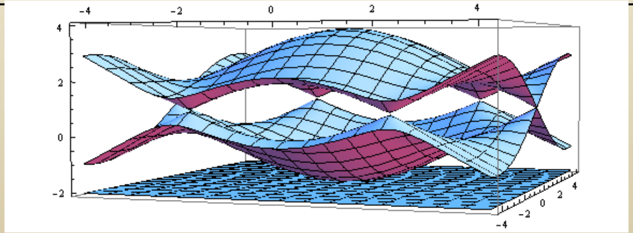
Due to DESTRUCTIVE INTERFERENCE !

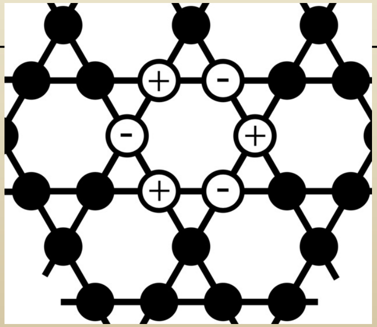
The CLS set can be:

- **orthonormal ➔ spans the entire FB Hilbert space**
- **linearly independent ➔ spans the entire FB Hilbert space**
- **linearly dependent ➔ missing states: CLLines, band touching**

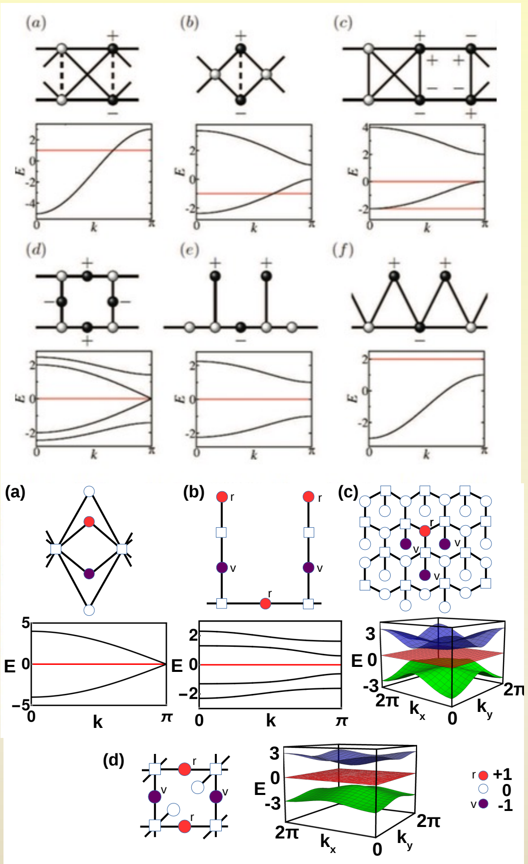
Certain fine-tunings allow for ALL BANDS FLAT (ABF) – orthonormal!

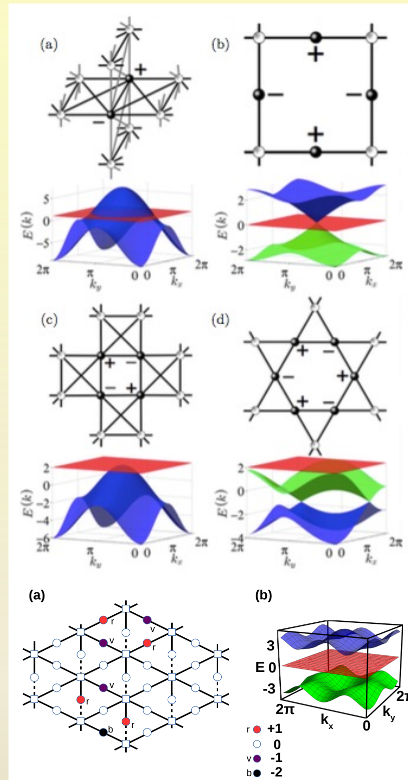


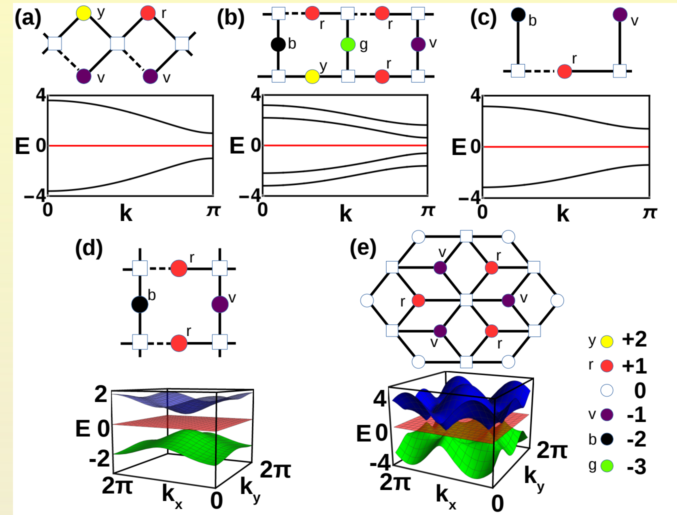


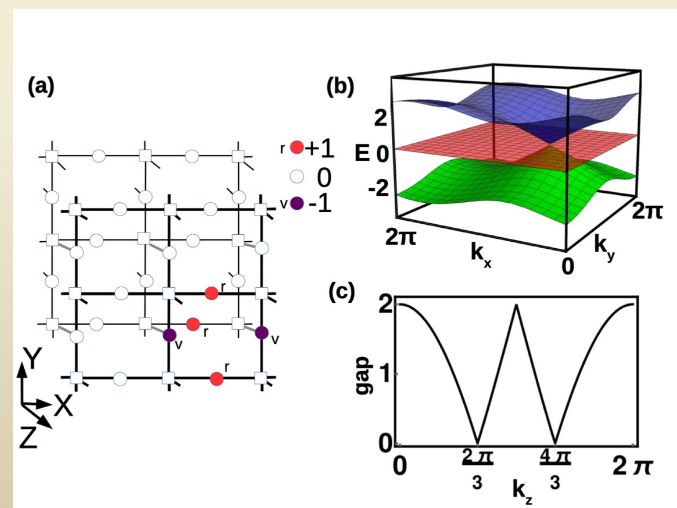


Compact Localized States in FB Hamiltonians

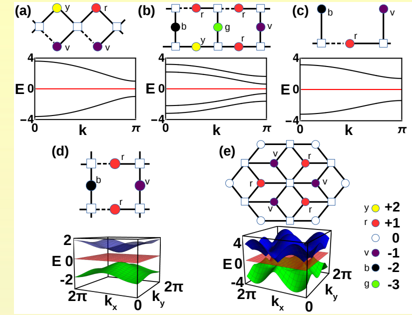


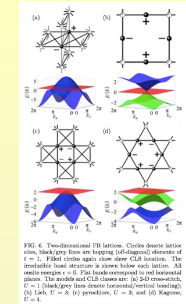


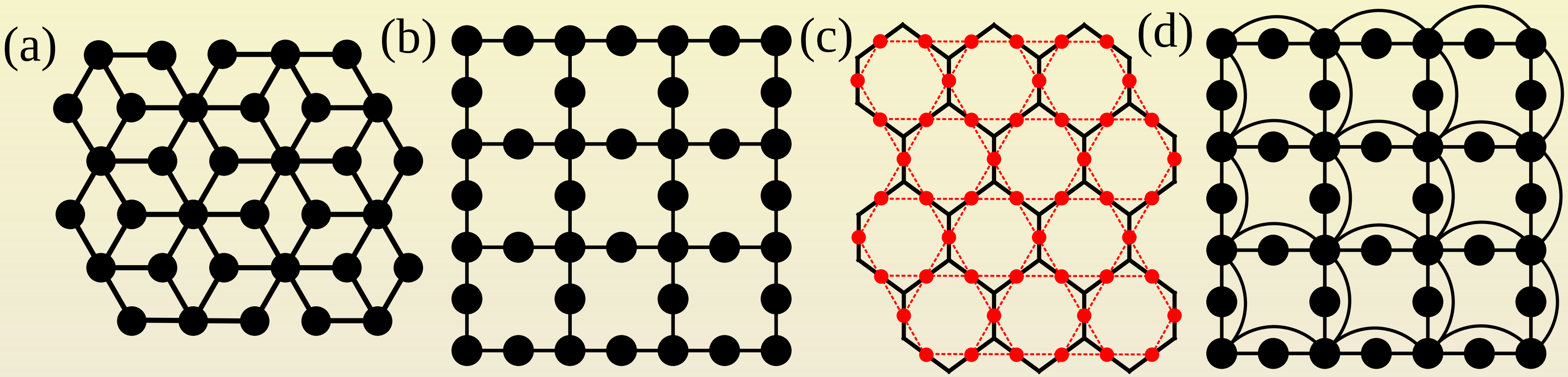




A bit on history







B. Sutherland 1986
CLS in dice lattice!

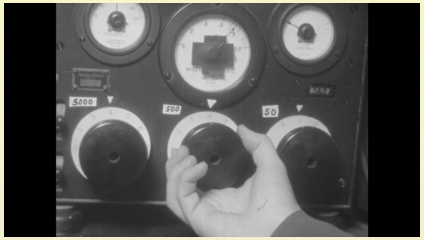
E.H.Lieb 1989
HTSC, CoO_2 planes
Hubbard model
Bipartite lattice
FB ferromagnetism

A.Mielke 1991
Line graphs!

H.Tasaki 1991
Decorate!
Perturb!
Keep FBs!

CLS-based FB generator

- A black box, with knobs
- Turn the knobs to the right position of FB classification
- Then turn the handle
- The box will produce all FB Hamiltonians



The knobs:

- Lattice dimension : d
- Number of bands : v , aka number of lattice sites per unit cell
- Unit cell size of CLS : U
- Shape of CLS : s (for d>1)

SF, D.Leykam, J.Bodyfelt, P.Matthies, A.S.Desyatnikov, EPL 105, 30001 (2014)

W.Maimaiti, A.Andreanov, H.C.Park, O. Gendelman, SF, PRB 95, 115135 (2017)

W. Maimaiti, SF, A. Andranov, PRB 99, 125129 (2019)

W. Maimaiti, A. Andreanov, SF, PRB 103, 165116 (2021)

W. Maimaiti, A. Andreanov, PRB 104, 035115 (2021)

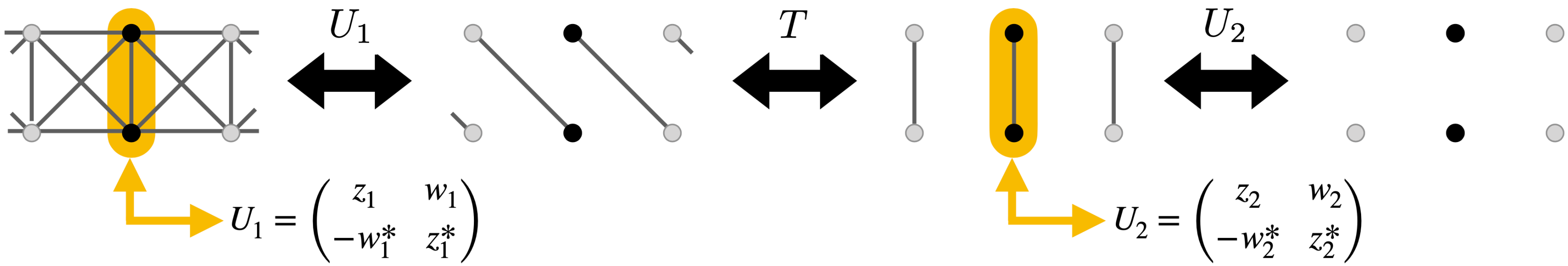
All Bands Flat

finite hopping range and number of bands, All Bands Flat: orthonormal CLS

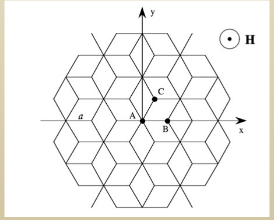
→ a finite number of noncommuting local unitaries will diagonalize H

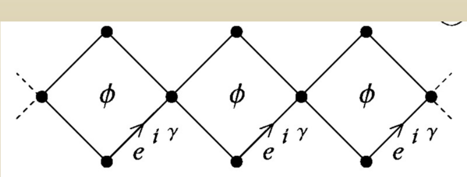
Example: $\nu = 2$ all band flat lattice

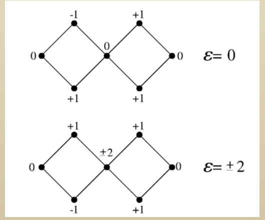
$$i\dot{\psi}_n = -H_0\psi_n - H_1\psi_{n+1} - H_1^{\dagger}\psi_{n-1}, \qquad e.g.\ \text{Creutz} \quad H_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$



historical examples: Vidal et al PRL 85 3906 (2000), PRL 81 5888 (1998)







Antisymmetries

Flatbands can also emerge as a consequence of a symmetry. Local and latent symmetries have been shown to generate flatbands [32,33]. The other class of symmetries are global symmetries of the Hamiltonian. A global symmetry is associated with a symmetry operator Γ which is either unitary or antiunitary. A single-particle Hamiltonian $\mathcal{H}$ is antisymmetric if the following relation holds: $\Gamma \cdot \mathcal{H} \cdot \Gamma^{-1} = -\mathcal{H}$. The antisymmetry implies that for each eigenvalue E with eigenvector $|\psi_E\rangle$ there exists the negative eigenvalue $-E$ with eigenvector $\Gamma\,|\psi_E\rangle$. If the total number of eigenvalues is odd, it follows that at least one of them is zero. Translationally invariant lattice Hamiltonians are characterized by the number of their sublattices. Transforming the Hamiltonian into Bloch momentum space and observing $\Gamma(\vec{k}) \cdot \mathcal{H}(\vec{k}) \cdot \Gamma^{-1}(\vec{k}) = -\mathcal{H}(\vec{k})$ results in a macroscopically degenerated symmetry-protected $E = 0$ flatband for an odd number of sublattices.

One such example is the chiral symmetry that is realized by a unitary operator Γ. The chiral Hamiltonian in momentum space turns bipartite, $\mathcal{H}(\vec{k}) = \begin{pmatrix} \mathbb{O} & \mathbb{T}(\vec{k}) \\ \mathbb{T}^\dagger(\vec{k}) & \mathbb{O} \end{pmatrix}$, where $\mathbb{O}$ is a null matrix and $\mathbb{T}(\vec{k})$ is a rectangular matrix. Chiral flatband models and exhausting flatband generators have been reported

Ramachandran,Andreanov,SF, PRB96 161104(R) 2017

Symmetry protected FB generators: Chiral Flat Bands

Bipartite Lattices Possess Chiral (Sublattice) Symmetry

- the lattice separates into A and B sublattices
- onsite energies are zero
- hoppings connect only A with B sublattices
- hoppings within each sublattice are absent

That is chiral symmetry

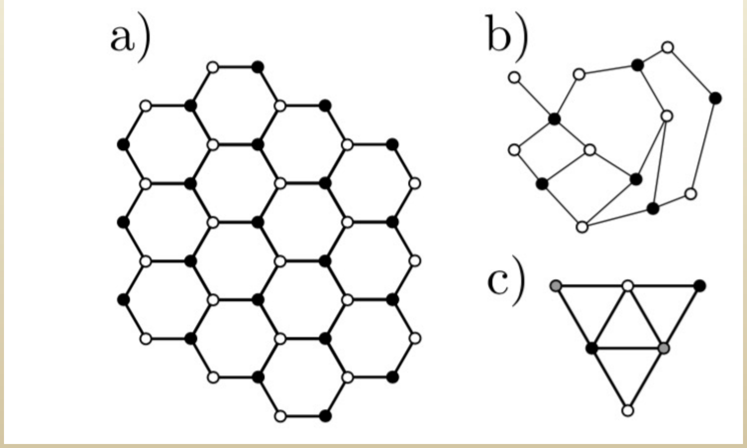
$$E\Psi_i^A = \sum t_{i,j}\Psi_j^B$$

$$E\Psi_i^B = \sum t_{i,j}\Psi_j^A$$

if $\{\Psi^A, \Psi^B\}$ is eigenstate to energy E

then $\{\pm\Psi^A, \mp\Psi^B\}$ is eigenstate to energy $-E$

$$H = \begin{pmatrix} 0 & T \\ T^\dagger & 0 \end{pmatrix}$$



Lieb's theorem (Lieb PRL 1989):

if dimension d^A is larger than d^B

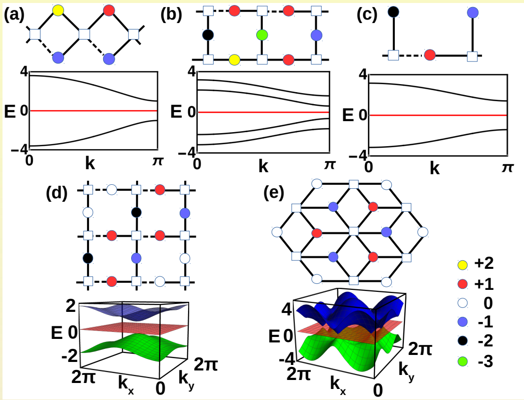
then there exist at least $(d^A - d^B)$ eigenstates

$\{\Psi^A, 0\}$ with energy $E = 0$.

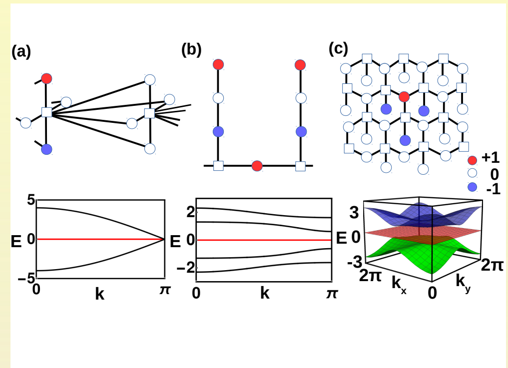
Ramachandran,Andreanov,SF, PRB96 161104(R) 2017

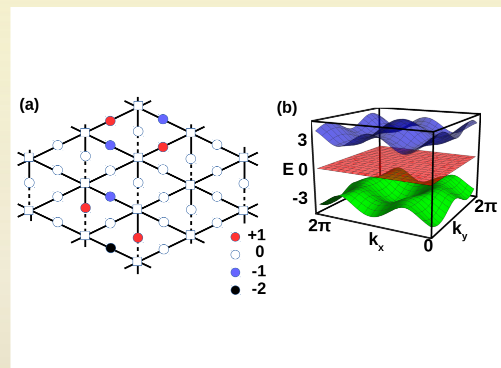
Symmetry protected FB generators: Chiral Flat Bands

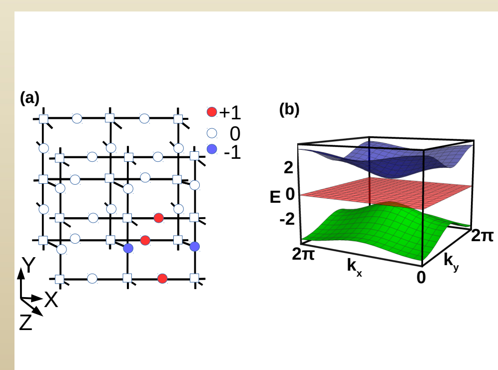
Known but modified CFBs



Generating new CFBs







Chiral Consequences

the CLS is always located on the majority sub lattice

assume that the CLS has a volume V
the number of equations $n_e \sim eV + sV^{1-1/d}$
the number of variables $n_v \sim vV$
for large enough V a CLS can be always computed

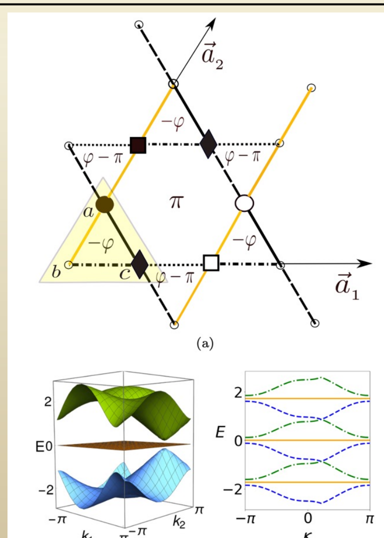
$\nu = 3:$ CLS with $U = 1$ exceptional even in $d = 1$
$\nu = 5:$ CLS with $U = 1$ only for $d = 1$ and $\mu^A = 4$

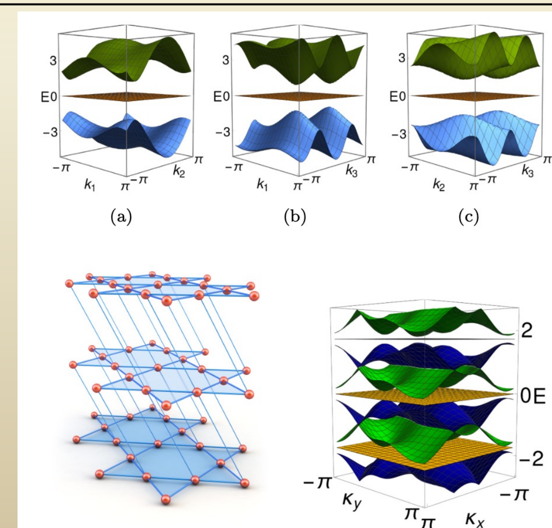
This is in particular true for any symmetry-preserving hopping disorder: Chiral flat bands are protected by symmetry, and their modified CLS as well!

PHYSICAL REVIEW A 105, L021305 (2022)
Letter
Anti-$\mathcal{PT}$ flatbands
Arindam Mallick,[1,*] Nana Chang,[1,2,3,†] Alexei Andreanov,[1,4,‡] and Sergej Flach[1,4,§]

anti-PT protection

- d-dimensional tight binding non-Bravais lattice, with odd number of bands
- anti-PT symmetry: $\mathcal{A}\cdot\mathcal{H}\cdot\mathcal{A}^{-1} = -\mathcal{H}$. $\mathcal{A} = \mathcal{T}\cdot\mathcal{P}$ ➔ central band E=0 is flat!
- This symmetry protected FB supports compact localized states
- Add DC field: infinite Wannier-Stark ladder of (d-1)-dimensional irreducible band structures
- Each irreducible band structure keeps one flat band!
- No compact localized states! Only exponential localization!





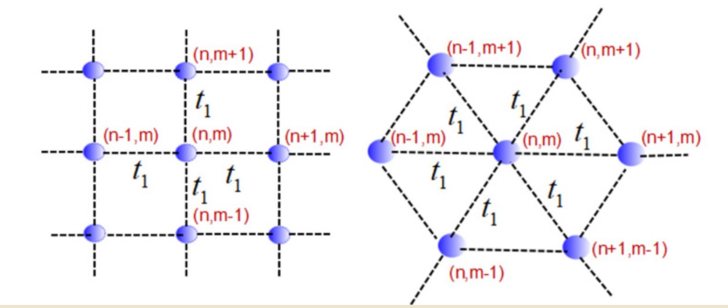
PHYSICAL REVIEW RESEARCH 3, 013174 (2021)

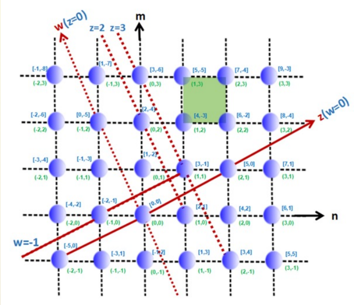
Wannier-Stark flatbands in Bravais lattices

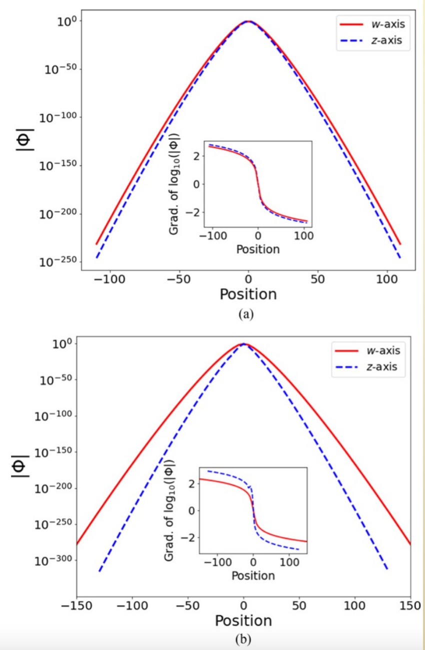
Arindam Mallick,[1,*] Nana Chang,[1,2] Wulayimu Maimaiti,[1,3] Sergej Flach,[1,4] and Alexei Andreanov[1,4,†]

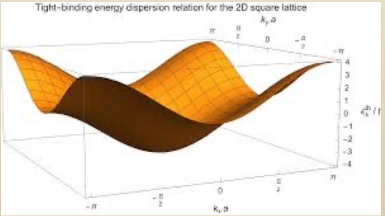
Wannier-Stark FBs: all different!

- d-dimensional tight binding Bravais lattice: one dispersive band
- Add DC field: infinite Wannier-Stark ladder of (d-1)-dimensional FBs!
- No compact localized states! Superexponential Localization!
- Only condition: no hoppings aligned along the direction perp to the DC!



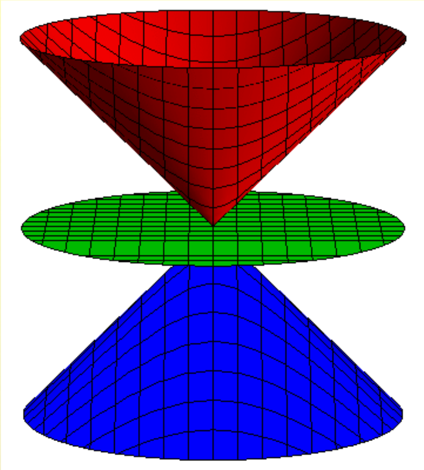






$$\mathcal{H} = \sum_{n,m} \Big[-\sum_{l,j} t_{lj} |n-l, m-j\rangle\langle n, m| + \vec{\mathcal{E}} \cdot \vec{r}_{nm} |n, m\rangle\langle n, m| \Big],$$

$$\lambda_E = a \in \mathbb{Z} \Rightarrow E = \mathcal{F}a.$$



+ perturbation = ?

Single particle perturbations (linear wave equations):

- CLS hybridization with dispersive states for non-gapped FB
- CLS-CLS hybridization (e.g. for gapped FB, or AllBandsFlat!)

Many body perturbations (linear, but high dimensional Hilbert space):

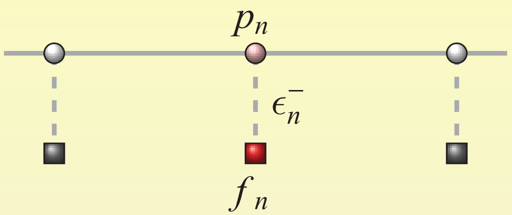
- critical filling factors for fermions due to Pauli for U>1
- interactions destroy ground state degeneracy
- Interaction induced transport

Mean field perturbations (nonlinear wave equations):

- persistence of CLS as periodic orbits
- enhancement of transport due to interactions

S. Flach et al, EPL (2014); D. Leykam et al, EPJB (2017)

Weak Disorder

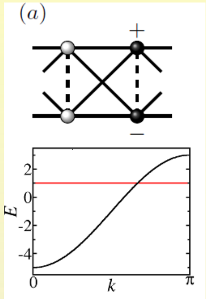


Non-gapped FB:
CLS represent strong scatterers
Effective Cauchy disorder for dispersive states
Novel scaling law at FB energy

Gapped FB:
Finite localization length due to CLS-CLS hybridization
Finite localization length due to gap separating from dispersive bands

Chiral FB with symmetry-preserving disorder:
CFB is preserved, and protected by nonzero gaps
CLS modify but stay compact

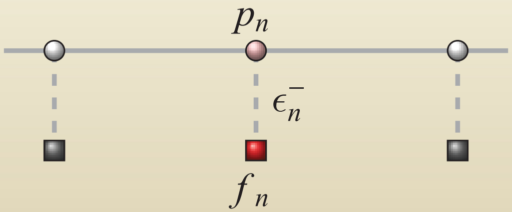
Example: Cross-Stitch



$$E\,a_n = \epsilon_n^a a_n - a_{n+1} - a_{n-1} - b_{n-1} - b_{n+1} - t\,b_n$$
$$E\,b_n = \epsilon_n^b b_n - a_{n+1} - a_{n-1} - b_{n-1} - b_{n+1} - t\,a_n$$

$$\epsilon_n^a = \epsilon_n^b = 0 \qquad E_{FB} = t, \quad E(k) = -4\cos(k) - t$$

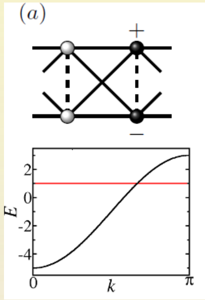
$$p_n = \frac{1}{\sqrt{2}}(a_n + b_n), \qquad f_n = \frac{1}{\sqrt{2}}(a_n - b_n),$$
$$\epsilon_n^+ = \frac{1}{2}(\epsilon_n^a + \epsilon_n^b), \qquad \epsilon_n^- = \frac{1}{2}(\epsilon_n^a - \epsilon_n^b),$$

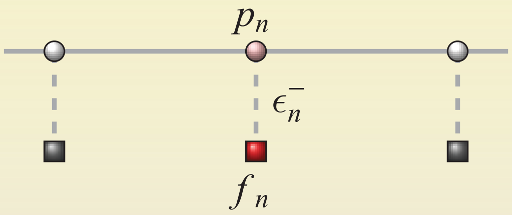


$$E\,p_n = (\epsilon_n^+ - t)\,p_n + \epsilon_n^- f_n - 2\,(p_{n+1} + p_{n-1})$$
$$E\,f_n = (\epsilon_n^+ + t)\,f_n + \epsilon_n^- p_n\ .$$

S. Flach et al, EPL (2014)

Treating Cross-Stitch





$$\mathcal{P}(\epsilon_n) = 1/W \text{ for } |\epsilon_n| \leq W/2$$

$$\left[E + t - \epsilon_n^+ - \frac{\left(\epsilon_n^-\right)^2}{E - t - \epsilon_n^+}\right] p_n = -2\left(p_{n-1} + p_{n+1}\right)$$

$$z = 1/\epsilon_n^+ \qquad \mathcal{W}(z) = \frac{1}{z^2}\int \mathcal{P}\left(\frac{1}{z}\right)\mathcal{P}\left(y - \frac{1}{z}\right) dy$$

CLS generate disorder with heavy Cauchy tails

Tight binding chain with Cauchy disorder:

$\xi \sim 1/W^{1}$ inside the spectrum

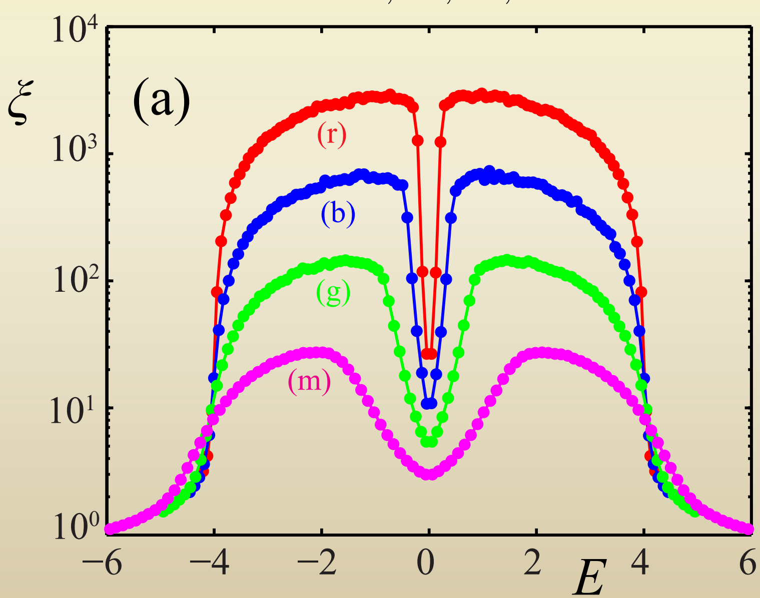
$\xi \sim 1/W^{1/2}$ at the edge

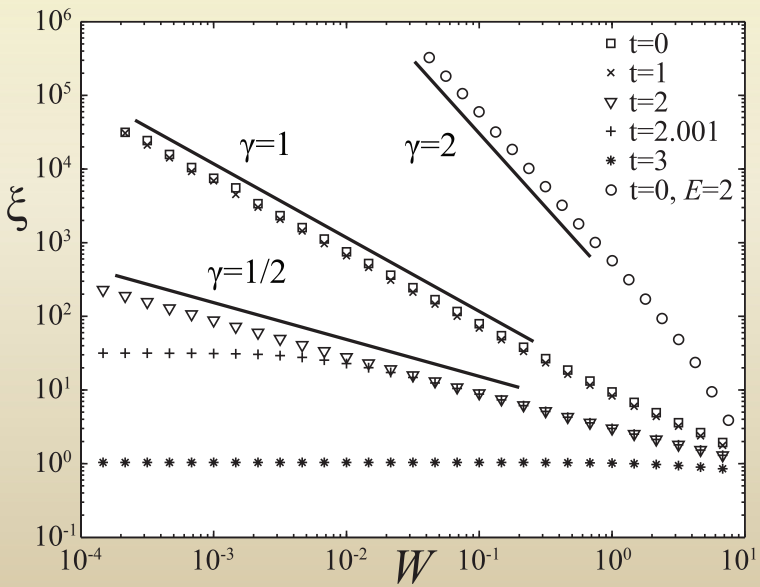
Lloyd '69,
Thouless '72
Ishii '73
Deych et al '00,'01
Titov et al '03

Treating Cross-Stitch

S. Flach et al, EPL (2014)

$W = 0.5\ ,\ 1\ ,\ 2\ ,\ 4$





$$\text{Gapped FB with } U = 1 : \ \xi(W \to 0) \to \text{constant} \neq 0$$

Bodyfelt,Leykam,Danieli,Yu,SF, PRL 2014
Danieli,Bodyfelt,SF, PRB 2015

1D cross-stitch, Aubry-Andre model

$$\epsilon_n^a = -\epsilon_n^b = \lambda \cos(2\pi\alpha n)$$

$$(E-t)\,p_n = \frac{(\epsilon_n^-)^2}{E+t}\,p_n - 2(p_{n-1}+p_{n+1})$$

$$(\epsilon_n^-)^2 = \lambda^2\cos^2(2\pi\alpha n) = \frac{\lambda^2}{2}\,[1+\cos(4\pi\alpha n)]$$

$$\tilde{E}\,p_n = \frac{\lambda^2}{4(E+t)}\cos(4\pi\alpha n) - (p_{n-1}+p_{n+1})$$

$$\tilde{E} := \frac{E-t}{2} - \frac{\lambda^2}{4(E+t)}$$

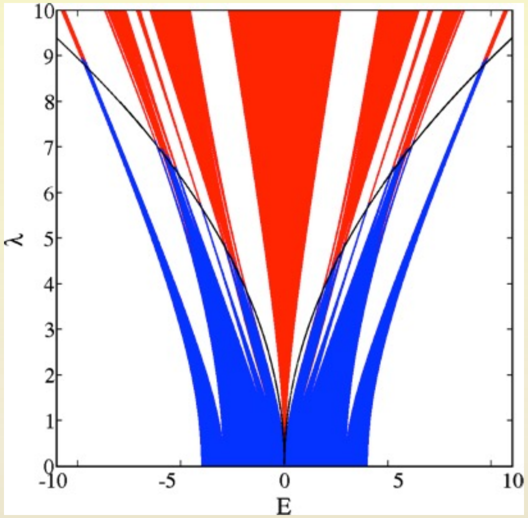
Mobility edge (energy dependent potential strength at metal-insulator transition)

$$\left|\frac{\lambda^2}{4(E+t)}\right| = 2 \;\Rightarrow\; \lambda(E) = 2\sqrt{2|E+t|}$$

Bodyfelt,Leykam,Danieli,Yu,SF, PRL 2014
Danieli,Bodyfelt,SF, PRB 2015

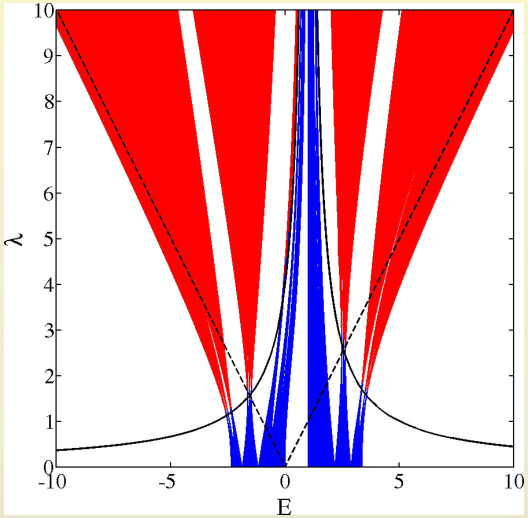
Quasiperiodic perturbations: engineering MIT and mobility edges!

Cross-stitch



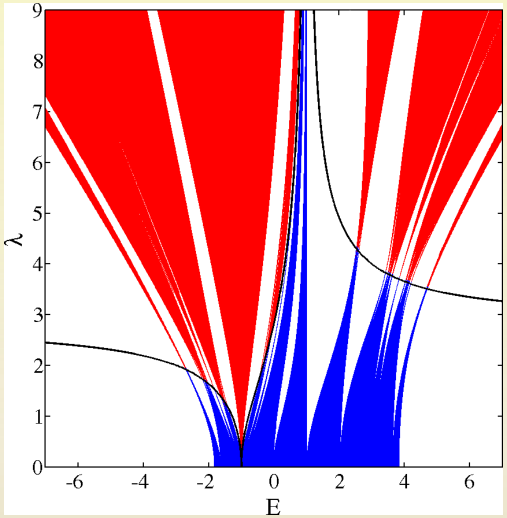
$$\lambda_c(E_c) = 2\sqrt{2|E_c - t|}$$

diamond



$$\lambda_c(E_c) = \left|\frac{4}{E_c - K}\right|$$

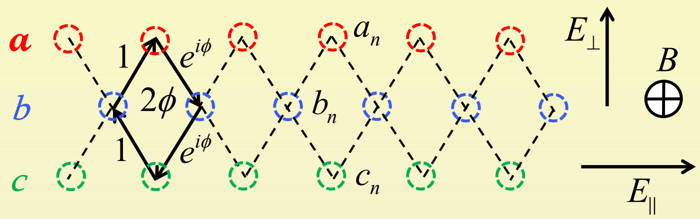
modified diamond

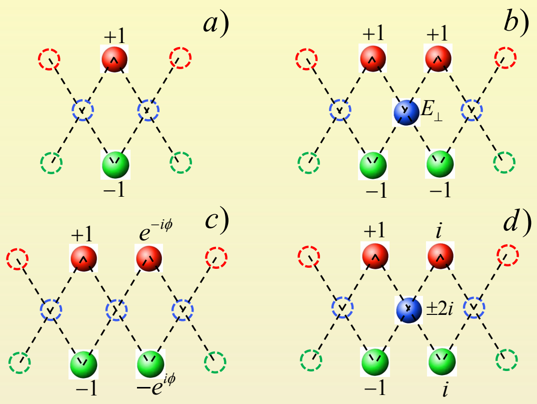


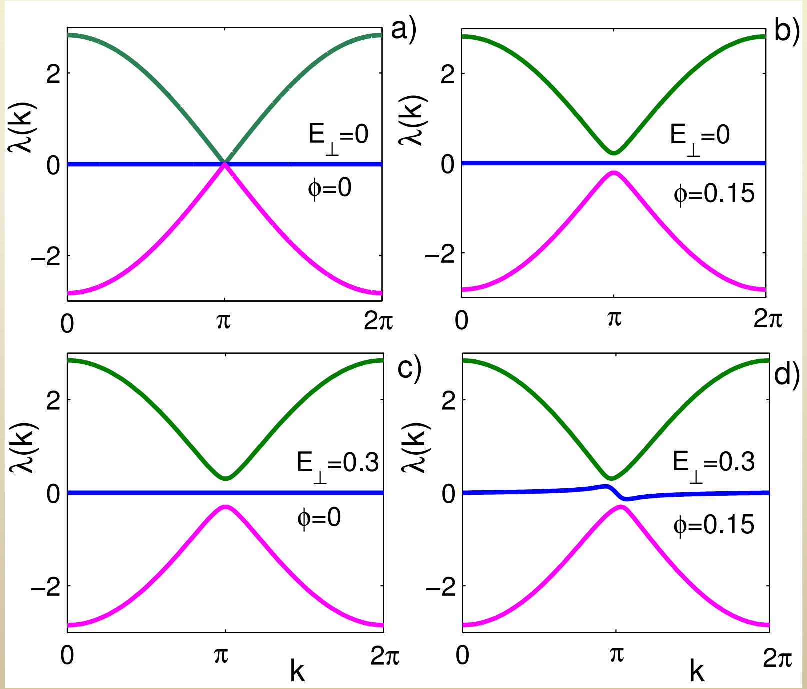
$$\lambda_c(E) = 2\sqrt{2\left|\frac{E - t}{E - K}\right|}$$

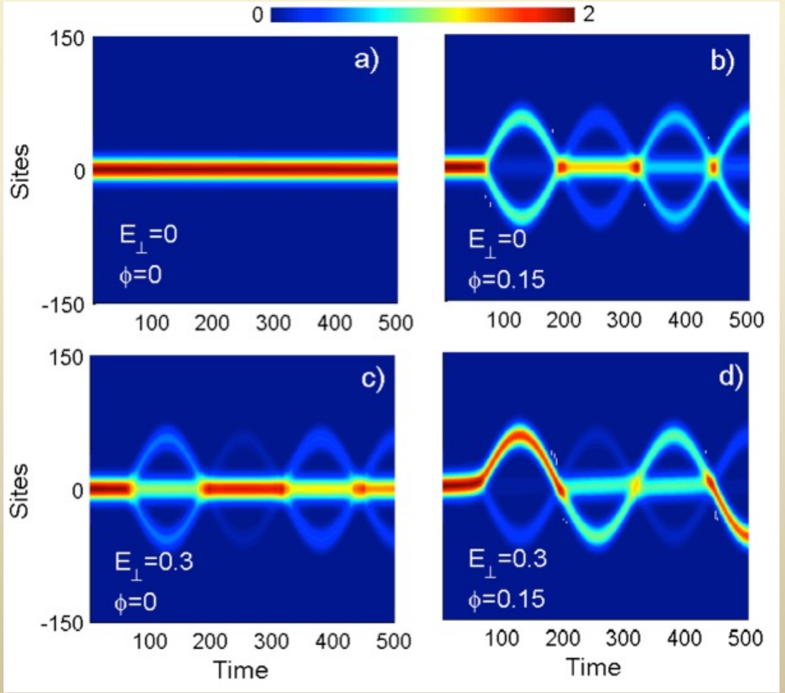
Khomeriki,SF, PRL 116 24301 (2016)

Landau-Zener Bloch oscillations with perturbed flat bands









All Bands Flat + Disorder ➔ Metal-Insulator Transitions

Letter

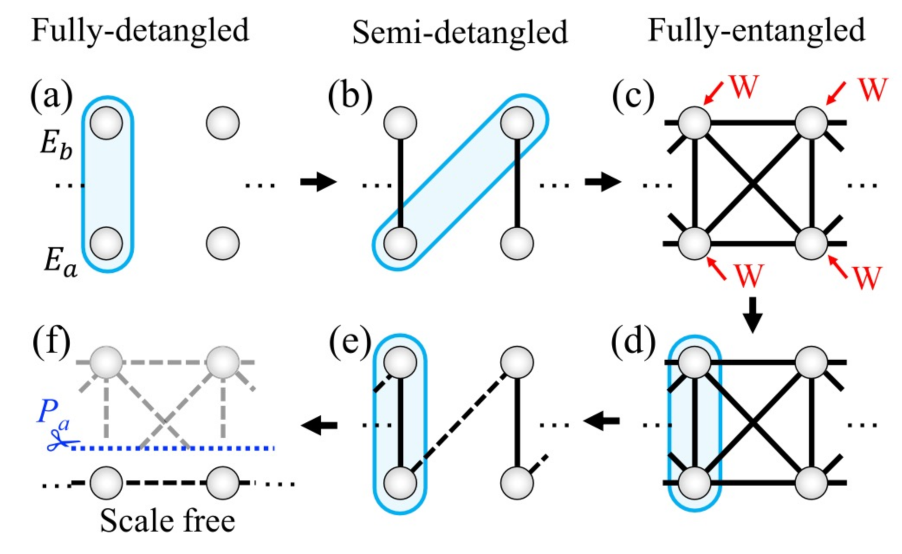
Metal-insulator transition in infinitesimally weakly disordered flat bands

Tilen Čadež *
Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Republic of Korea

Yeongjun Kim,† Alexei Andreanov,‡ and Sergej Flach§
Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Republic of Korea and Basic Science Program, Korea University of Science and Technology (UST), Daejeon 34113, Republic of Korea

(Received 26 July 2021; revised 29 October 2021; accepted 2 November 2021; published 15 November 2021)

We study the effect of infinitesimal onsite disorder on d-dimensional all bands flat lattices. The lattices are



$$\mathcal{H} = \mathcal{U}\mathcal{H}_{\mathrm{FD}}\mathcal{U}^\dagger$$

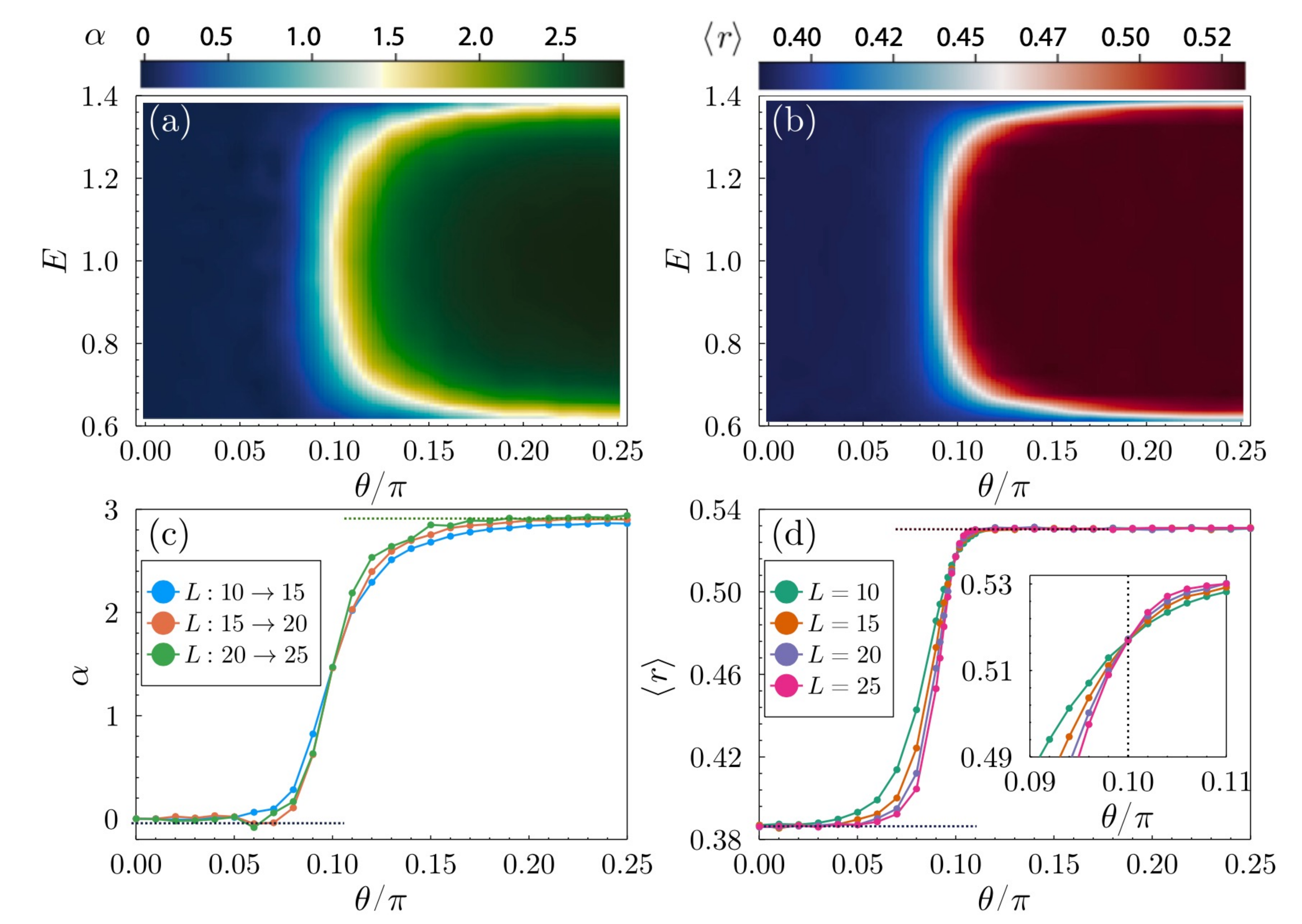
$$\mathcal{H}_d = W D$$

$$\mathcal{H}_{\mathrm{tot}} = \mathcal{H}_{\mathrm{FD}} + W\,\mathcal{U}^\dagger D \mathcal{U}$$

$$\mathcal{H}_P = W\,\mathcal{H}_{\mathrm{sf}}$$

$$\mathcal{H}_{\mathrm{sf}} = P_a \mathcal{U}^\dagger D \mathcal{U} P_a$$

All Bands Flat + Disorder ➔ Metal-Insulator Transitions in d=3



All Bands Flat + disorder potentials ➔ metal-insulator transitions

PHYSICAL REVIEW B **104**, L180201 (2021)

Letter

Metal-insulator transition in infinitesimally weakly disordered flat bands

Tilen Čadež*

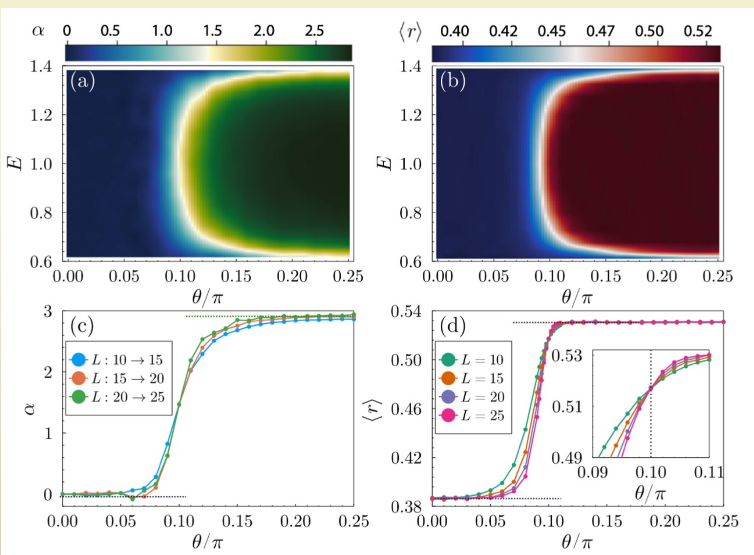
Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Republic of Korea

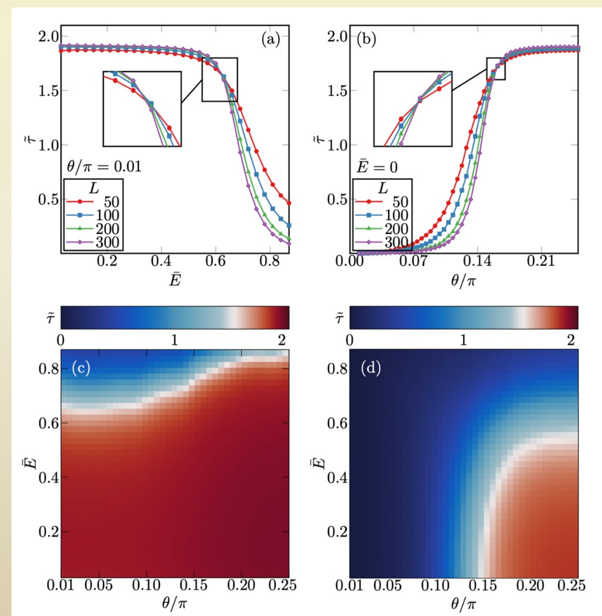
Yeongjun Kim,† Alexei Andreanov,‡ and Sergej Flach§

PHYSICAL REVIEW B **107**, 174202 (2023)

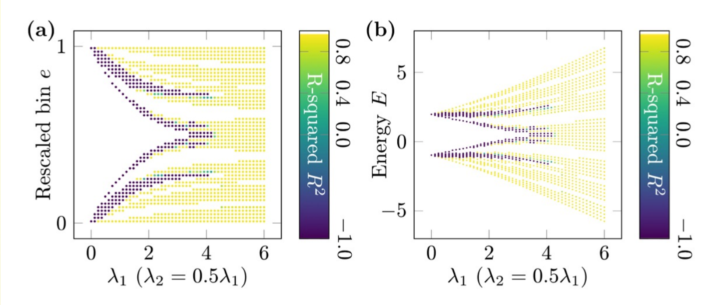
Flat band induced metal-insulator transitions for weak magnetic flux and spin-orbit disorder

Yeongjun Kim[1,2,*] Tilen Čadež,[1,†] Alexei Andreanov[1,2,‡] and Sergej Flach[1,2,§]





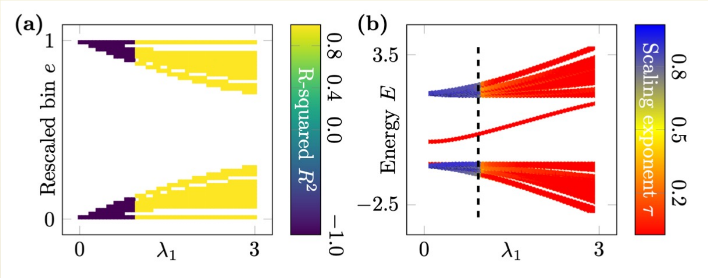
All Bands Flat + quasiperiodic potentials ➔ Fractality Edges in d=1



PHYSICAL REVIEW B **107**, 014204 (2023)

Critical-to-insulator transitions and fractality edges in perturbed flat bands

Sanghoon Lee, Alexei Andreanov, and Sergej Flach



Chaos ARTICLE pubs.aip.org/aip/cha

Critical state generators from perturbed flatbands

Cite as: Chaos 33, 073125 (2023); doi: 10.1063/5.0153819
Submitted: 12 April 2023 · Accepted: 19 June 2023 ·
Published Online: 11 July 2023

View Online Export Citation CrossMark

S. Lee,[1,2,a] S. Flach,[1,2,b] and Alexei Andreanov[1,2,c]

Experiments: FBs !

2012

New Journal of Physics
The open-access journal for physics

Exciton–polariton condensates with flat bands in a two-dimensional kagome lattice

Naoyuki Masumoto[1,2,3,6], Na Young Kim[4], Tim Byrnes[2], Kenichiro Kusudo[2,3], Andreas Löffler[5], Sven Höfling[5], Alfred Forchel[5] and Yoshihisa Yamamoto[1,2,4]

2014

New Journal of Physics
The open access journal at the forefront of physics
Deutsche Physikalische Gesellschaft DPG | IOP Institute of Physics

Experimental observation of bulk and edge transport in photonic Lieb lattices

D Guzmán-Silva[1,4], C Mejía-Cortés[1], M A Bandres[2], M C Rechtsman[2], S Weimann[3], S Nolte[3], M Segev[2], A Szameit[3] and R A Vicencio[1]

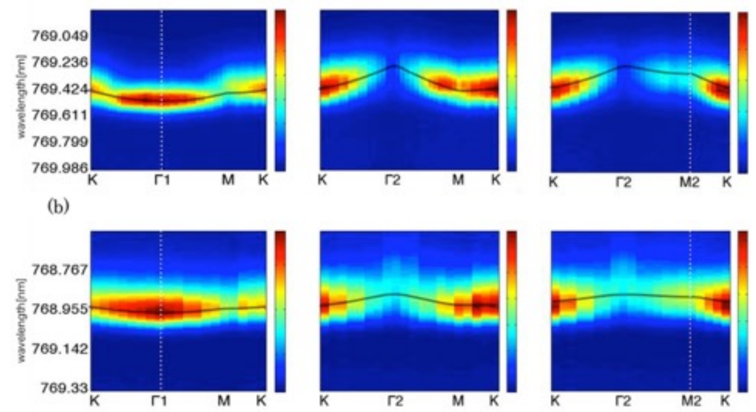


Figure 5. Measured energy dispersions along the high-symmetry points of the first (left), second (middle) and third (right) BZs within $a = 3\,\mu$m (a) and $6\,\mu$m (b) kagome lattice devices. Black transparent lines are the calculated band structures.

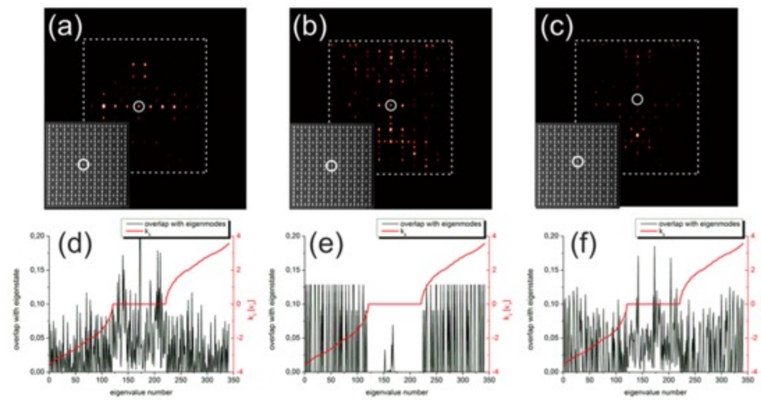
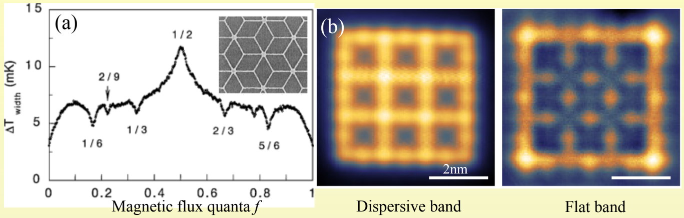
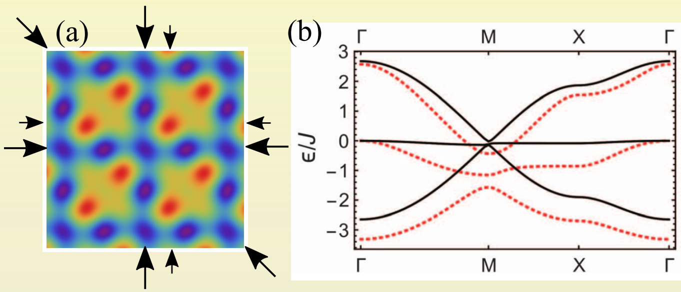


Figure 2. Experimentally observed light distribution at the output facet of the lattice, when exciting (a) the *A*-site, (b) the *B*-site and (c) the *C*-site. The overlap with the eigenmodes in the three bands is shown in (d) for the *A*-site, (e) for the *B*-site and (f) for the *C*-site. It is evident that transport is stronger when the flat band is least populated at the initial plane of excitation.

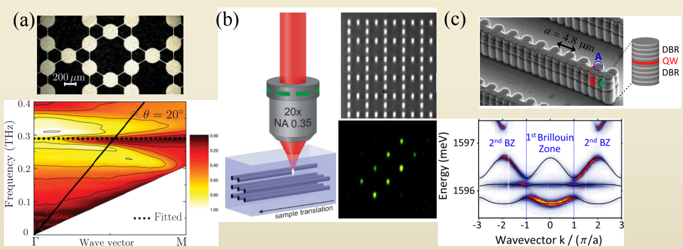
Compact Localized States in FB Hamiltonians : experiments



(a) Aharonov-Bohm cage in a superconducting wire network.
The superconducting transition width shows a broad peak
in the flat band limit *f* = ½ due to compact localized states.
C.C.Abilio,P.Butaud,T.Fournier,B.Pannetier,J.Vidal,S.TedescoandB.Dalzotto
Phys. Rev. Lett. 83 (1999) 5102
(b) Selective imaging of dispersive and flat band states of an electronic Lieb lattice.
R. Drost, T. Ojanen, A. Harju and P. Liljeroth, Nat. Phys. 13 (2017), 668

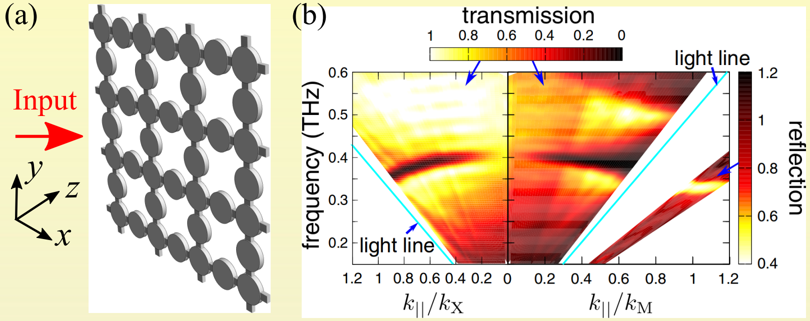


An optical Lieb lattice for cold atoms. Lattice potential formed by five interfering standing waves.Lattice sites lie at the potential minima in blue.
Band structure for shallow (red dashed lines) and deep (solid black lines) lattices.
S. Taie, H. Ozawa, T. Ichinose, T. Nishio, S. Nakajima and T. Takahashi, Sci. Adv. 1 (2015)

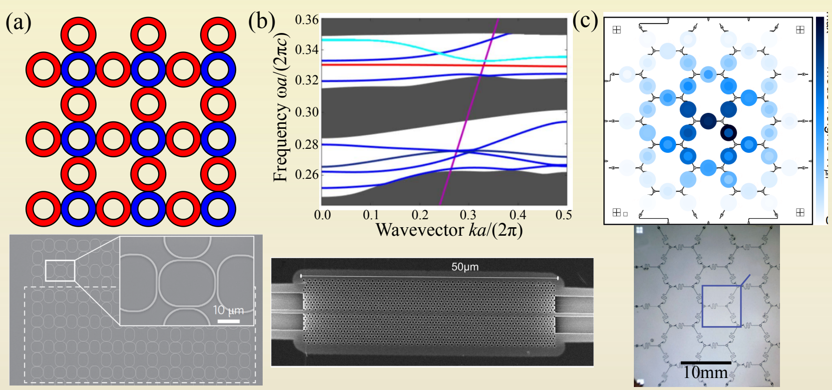


Examples of photonic flat bands.
(a) Kagome lattice for terahertz spoof plasmons, displaying an omnidirectional minimum in the transmission at the flat band frequency (dashed line).
Y. Nakata, T. Okada, T. Nakanishi and M. Kitano, Phys. Rev. B 85 (2012) 205128
(b) Femtosecond laser-written Lieb lattice waveguide arrays and an observed compact localized flat band state.
R.A. Vicencio, C. Cantillano, L. Morales-Inostroza, B. Real, C. Mejía-Cortés, S. Weimann, A. Szameit and M.I. Molina, Phys. Rev. Lett. 114 (2015) 245503
(c) Structured microcavity forming a 1D stub lattice and its photoluminescence spectrum revealing a middle flat band.
F. Baboux, L. Ge, T. Jacqmin, M. Biondi, E. Galopin, A. Lemaître, L. Le Gratiet, I. Sagnes, S. Schmidt, H.E. Türeci, A. Amo and J. Bloch, Phys. Rev. Lett. 116 (2016) 066402

Compact Localized States in FB Hamiltonians : experiments



Lieb lattice for spoof surface plasmons.
S. Kajiwara, Y. Urade, Y. Nakata, T. Nakanishi, and M. Kitano, Phys. Rev. B 93, 075126 (2016).



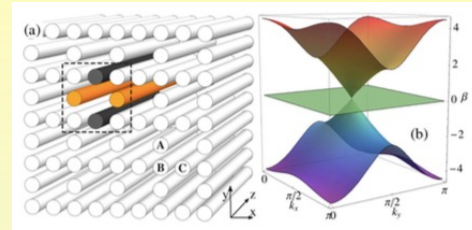
Novel platforms for exploring photonic flat bands.
(a) Coupled resonator lattices.
Top: Lieb-like lattice formed by detuned "site" (blue) and "link" (red) rings.
Bottom: scanning electron microscope image of the lattice.
M. Hafezi, S. Mittal, J. Fan, A. Migdall, and J. M. Taylor, Nature Photon. 7, 1001 (2013).
(b) Photonic crystals.
Top: Simulated band structure of 1D photonic crystal waveguide formed by removing a row of holes from kagome photonic crystal.
Bottom: Fabricated photonic crystal membrane.
S. A. Schulz, J. Upham, L. O'Faolain, and R. W. Boyd, Opt. Lett. 42, 3243 (2017)
(c) Microwave circuit QED.
Top: Space-resolved imaging of a dispersive band eigenstate of a kagome lattice of 49 coupled microwave resonators.
Bottom: Picture of the device.
D. L. Underwood, W. E. Shanks, A. C. Y. Li, L. Ateshian, J. Koch, and A. A. Houck, Phys. Rev. X 6, 021044 (2016)

Compact Localized States in FB Hamiltonians : experiments

Selected for a Viewpoint in *Physics*
PRL **114,** 245503 (2015) PHYSICAL REVIEW LETTERS week ending 19 JUNE 2015

Observation of Localized States in Lieb Photonic Lattices

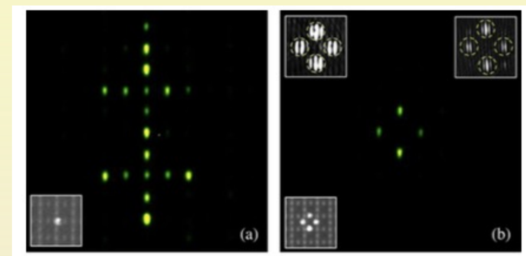
Rodrigo A. Vicencio,[1,*] Camilo Cantillano,[1] Luis Morales-Inostroza,[1] Bastián Real,[1] Cristian Mejía-Cortés,[1,†] Steffen Weimann,[2] Alexander Szameit,[2] and Mario I. Molina[1]
[1]*Departamento de Física, MSI-Nucleus on Advanced Optics, and Center for Optics and Photonics (CEFOP), Facultad de Ciencias, Universidad de Chile, Santiago 7800003, Chile*
[2]*Institute of Applied Physics, Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, 07743 Jena, Germany*



Selected for a Viewpoint in *Physics*
PRL **114,** 245504 (2015) PHYSICAL REVIEW LETTERS week ending 19 JUNE 2015

Observation of a Localized Flat-Band State in a Photonic Lieb Lattice

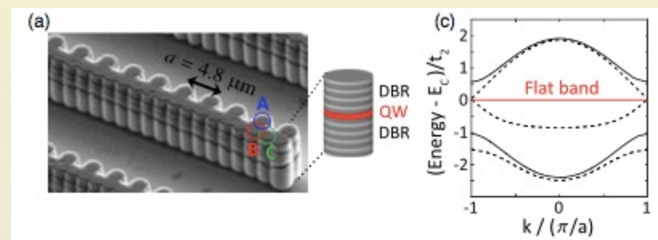
Sebabrata Mukherjee,[1,*] Alexander Spracklen,[1] Debaditya Choudhury,[1] Nathan Goldman,[2,3] Patrik Öhberg,[1] Erika Andersson,[1] and Robert R. Thomson[1]
[1]*SUPA, Institute of Photonics and Quantum Sciences, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom*
[2]*Center for Nonlinear Phenomena and Complex Systems, Université Libre de Bruxelles, CP 231, Campus Plaine, B-1050 Brussels, Belgium*
[3]*Laboratoire Kastler Brossel, Collège de France, 11 Place Marcelin Berthelot, 75005 Paris, France*



PRL **116,** 066402 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

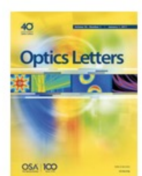
Bosonic Condensation and Disorder-Induced Localization in a Flat Band

F. Baboux,[1,*] L. Ge,[2,3] T. Jacqmin,[1,†] M. Biondi,[4] E. Galopin,[1] A. Lemaître,[1] L. Le Gratiet,[1] I. Sagnes,[1] S. Schmidt,[4] H. E. Türeci,[5] A. Amo,[1] and J. Bloch[1,6]
[1]*Laboratoire de Photonique et de Nanostructures (LPN), CNRS, Université Paris-Saclay, route de Nozay, F-91460 Marcoussis, France*
[2]*Department of Engineering Science and Physics, College of Staten Island, CUNY, New York 10314, USA*
[3]*The Graduate Center, CUNY, New York 10016, USA*
[4]*Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland*
[5]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*
[6]*Physics Deparment, Ecole Polytechnique, Université Paris-Saclay, F-91128 Palaiseau Cedex, France*

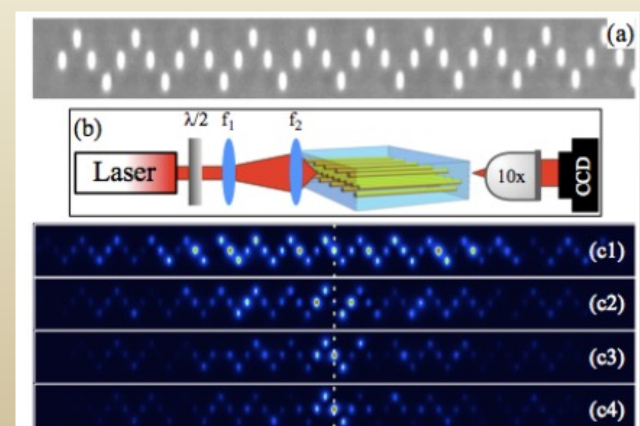


Transport in Sawtooth photonic lattices

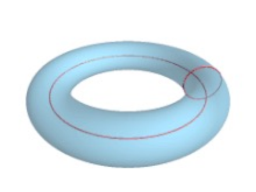
Steffen Weimann, Luis Morales-Inostroza, Bastián Real, Camilo Cantillano, Alexander Szameit, and Rodrigo A. Vicencio
Author Information ▾ Find other works by these authors ▾

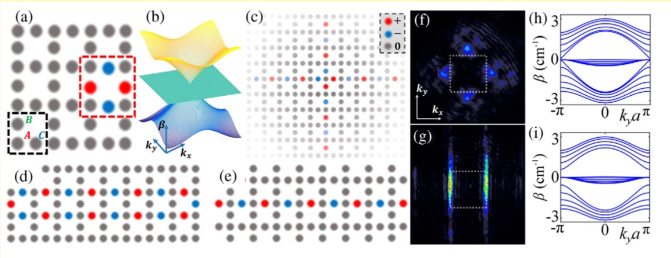


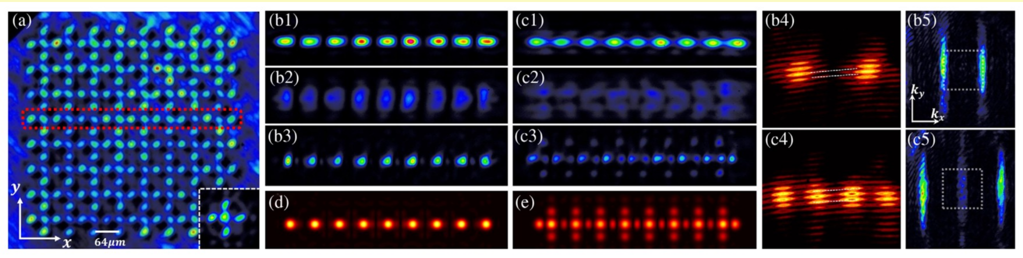
Optics Letters Vol. 41, Issue 11, pp. 2414-2417 (2016) • https://doi.org/10.1364/OL.41.002414

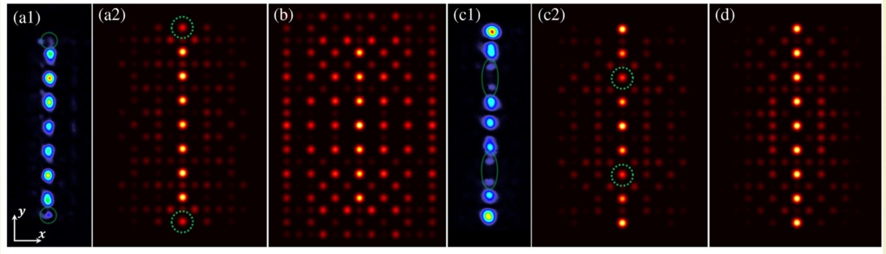


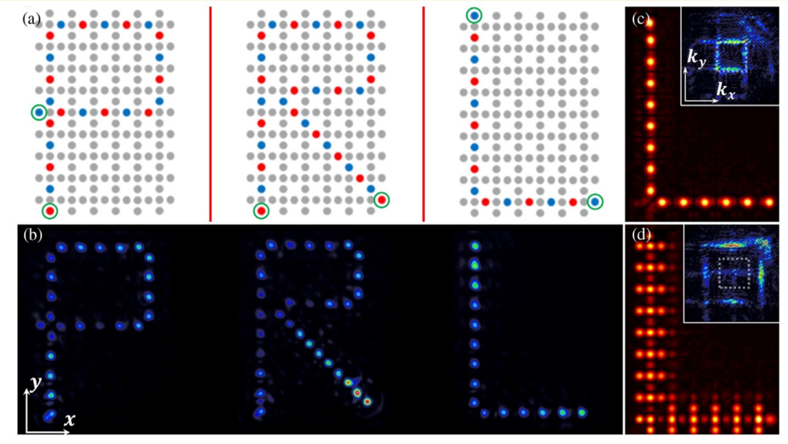
Compact Localized States in FB Hamiltonians : experiments











PHYSICAL REVIEW LETTERS **121,** 263902 (2018)

Unconventional Flatband Line States in Photonic Lieb Lattices

Shiqi Xia,[1] Ajith Ramachandran,[2,‖] Shiqiang Xia,[1] Denghui Li,[1] Xiuying Liu,[1] Liqin Tang,[1] Yi Hu,[1] Daohong Song,[1,3,*] Jingjun Xu,[1,3] Daniel Leykam,[2,†] Sergej Flach,[2,‡] and Zhigang Chen[1,3,4,§]

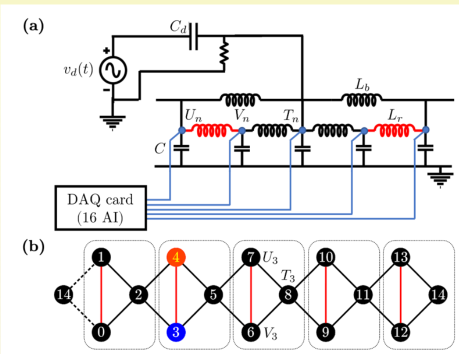
arXiv > cond-mat > arXiv:2307.15319

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 28 Jul 2023 (v1), last revised 3 Aug 2023 (this version, v2)]

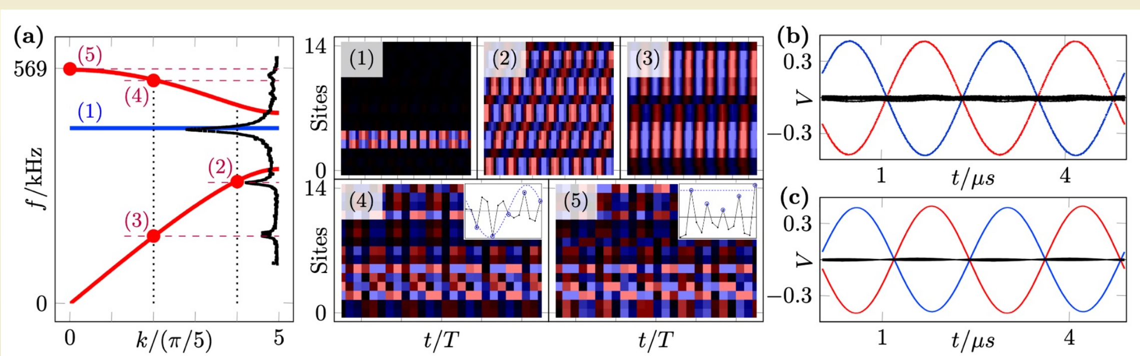
Compact Localized States in Electric Circuit Flatband Lattices

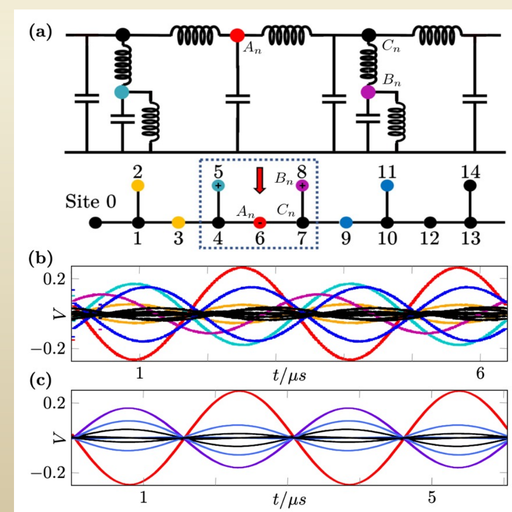
Carys Chase-Mayoral, L.Q. English, Yeongjun Kim, Sanghoon Lee, Noah Lape, Alexei Andreanov, P.G. Kevrekidis, Sergej Flach



$$\ddot{T}_n + \beta \dot{T}_n = -\omega_b^2 \left[4T_n - U_n - U_{n+1} - V_n - V_{n+1}\right]$$
$$\ddot{U}_n + \beta \dot{U}_n = -\omega_b^2 \left[(2+\alpha)\, U_n - \alpha V_n - T_n - T_{n-1}\right] \quad (1)$$
$$\ddot{V}_n + \beta \dot{V}_n = -\omega_b^2 \left[(2+\alpha)\, V_n - \alpha U_n - T_n - T_{n-1}\right].$$

At the driven site U_m, the rhs term in (1) gets a correction factor $1/(1+\gamma)$ and an additive driving force $A\sin(\omega_d t)$ (see [41] for details). Note that $\omega_b^2{=}1/(L_b C)$, $\gamma = C_d/C$, and $A = \gamma(1+\gamma)^{-1} v_d \omega_d^2$. Here, $\alpha = L_b/L_r$ is a tunable parameter that is able to shift the flatband, $\beta = R/L$ represents dissipation. Note that unlike the





flat band physics

2. many body flat band physics

Sergej Flach
Center for Theoretical Physics of Complex Systems
Institute for Basic Science
Daejeon South Korea

1. nonlinear CLS
2. many body flat band focalization
3. disorder free many body localization
4. trapping hard core bosons

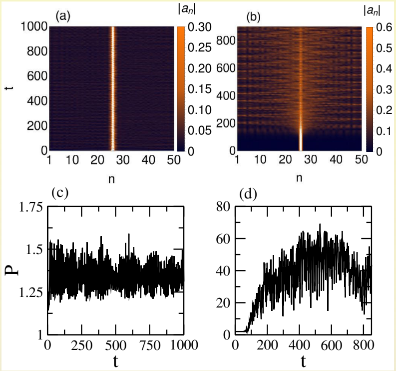
Adv Phys: X 3 1473052 (2018) ; APL PHOTONICS 3 070901 (2018)

Nonlinear Perturbations: Compact Discrete Breathers

Homogeneous CLS: continue into families of compact DBs

Inhomogeneous CLS: continue into families of usual DBs
additional finetuning: isolated compact DBs

Kevrekidis,Konotop (2002)
Johansson,Naether,Vicencio (2015)
Belicev,Gligoric,Maluckov,Stepic,Johansson (2017)
Danieli,Maluckov,Flach (2018)

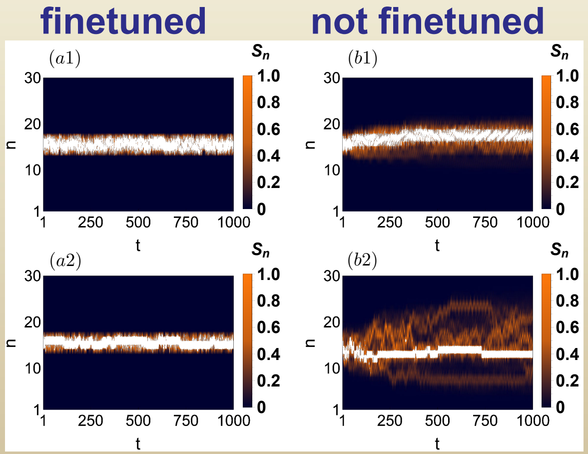


All Bands Flat & Nonlinear Perturbations: Compact Caging

Finetuning: FBs

$$i\dot{\psi}_n = -H_0\psi_n - H_1\psi_{n+1} - H_1^\dagger\psi_{n-1} + U\mathcal{F}(\psi_n)\psi_n,$$
$$\text{with}\quad \mathcal{F}(\psi_n) = \begin{pmatrix} |a_n|^2 & 0 \\ 0 & |b_n|^2 \end{pmatrix}.$$

More Finetuning: ABF



Even More Finetuning + Nonlinearity:
Caging of Any Compact Dynamical State

Danieli,Andreanov,Mithun,Flach PRB 104, 085131 (2021)

All Bands Flat + Interactions ➔ Many Body Flat Band Localization

Many-body interactions:

- restore transport in general
- two particle transport channels
- density assisted single particle transport channels

- Can we finetune interactions to prohibit transport?

PHYSICAL REVIEW B **102**, 041116(R) (2020)

Rapid Communications

Many-body flatband localization

Carlo Danieli,[1] Alexei Andreanov,[2,3] and Sergej Flach[2,3,4]

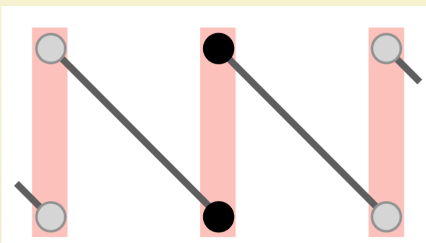
All Bands Flat + Interactions ➔ Many Body Flat Band Localization

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\text{sp}} + \hat{\mathcal{H}}_{\text{int}}, \qquad \hat{\mathcal{H}}_{\text{sp}} = \sum_k \hat{f}_k, \quad \hat{\mathcal{H}}_{\text{int}} = \sum_\kappa \hat{g}_\kappa$$

semi-detangled (SD) :

$$\hat{f}_k = \sum_{a,b=1}^{\nu} t_{ab}\hat{c}^\dagger_{k,a}\hat{c}_{k,b} + \text{H.c.}$$

$$\hat{g}_\kappa = \sum_{\alpha,\beta,\gamma,\delta=1}^{\nu} J_{\alpha\beta\gamma\delta}\hat{c}^\dagger_{\kappa,\alpha}\hat{c}^\dagger_{\kappa,\beta}\hat{c}_{\kappa,\gamma}\hat{c}_{\kappa,\delta} + \text{H.c.}$$



$$[\hat{f}_k, \hat{f}_{k'}] = [\hat{g}_\kappa, \hat{g}_{\kappa'}] = 0 \text{ for any } k, k', \kappa, \kappa'$$

$$[\hat{f}_k, \hat{g}_\kappa] \neq 0$$

If f=0 or g=0 : no charge transport, no energy transport

If not : it depends

Fully-detangled (FD):
f diagonal, g product of densities

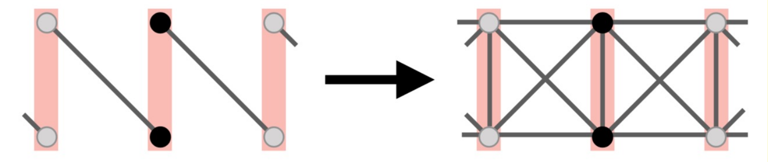
PHYSICAL REVIEW B **102**, 041116(R) (2020)

Rapid Communications

Many-body flatband localization

Carlo Danieli,[1] Alexei Andreanov,[2,3] and Sergej Flach[2,3,4]

All Bands Flat + Interactions ➔ Many Body Flat Band Localization



$$\hat{\mathcal{H}}_{\rm sp} = \sum_\kappa \hat{f}_\kappa = \sum_\kappa \left[\tfrac{1}{2}\hat{C}_\kappa^{\dagger T} H_0 \hat{C}_\kappa + \hat{C}_\kappa^{\dagger T} H_1 \hat{C}_{\kappa+1} + \text{H.c.}\right],$$

$$H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix}$$

$$J_{\alpha\beta\gamma\delta} = J_{\alpha\beta\alpha\beta}\delta_{\alpha,\gamma}\delta_{\beta,\delta}$$

$$J_{\alpha\beta\alpha\beta} = 1 \quad \alpha = \beta$$

$$J_{\alpha\beta\alpha\beta} = 2 \quad \alpha \neq \beta.$$

$$U_{ab}: \begin{cases} \hat{c}_{\kappa,a} = z\hat{d}_{\kappa,a} + w\hat{d}_{\kappa,b} \\ \hat{c}_{\kappa,b} = -w^*\hat{d}_{\kappa,a} + z^*\hat{d}_{\kappa,b} \end{cases}$$

$$H_0 = \begin{pmatrix} |z|^2 & z^*w \\ zw^* & |w|^2 \end{pmatrix}, \quad H_1 = t\begin{pmatrix} -z^*w^* & (z^*)^2 \\ -(w^*)^2 & z^*w^* \end{pmatrix}$$

$$\hat{\mathcal{H}}_{\rm int} = \sum_\kappa [\hat{a}_\kappa^\dagger\hat{a}_\kappa^\dagger\hat{a}_\kappa\hat{a}_\kappa + \hat{b}_\kappa^\dagger\hat{b}_\kappa^\dagger\hat{b}_\kappa\hat{b}_\kappa + 2\hat{a}_\kappa^\dagger\hat{a}_\kappa\hat{b}_\kappa^\dagger\hat{b}_\kappa] = \sum_\kappa [\hat{n}_{a,\kappa} + \hat{n}_{b,\kappa} - 1][\hat{n}_{a,\kappa} + \hat{n}_{b,\kappa}]$$

Local integrals of motion:

$$\hat{I}_\kappa = |z|^2(\hat{n}_{a,\kappa-1} + \hat{n}_{b,\kappa}) + |w|^2(\hat{n}_{a,\kappa} + \hat{n}_{b,\kappa-1}) + z^*w(\hat{a}_{\kappa-1}^\dagger\hat{b}_{\kappa-1} - \hat{a}_\kappa^\dagger\hat{b}_\kappa) + \text{H.c.}$$

PHYSICAL REVIEW B **102**, 041116(R) (2020)

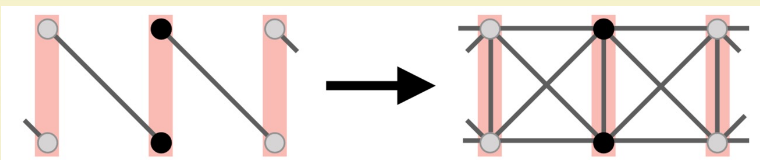
Rapid Communications

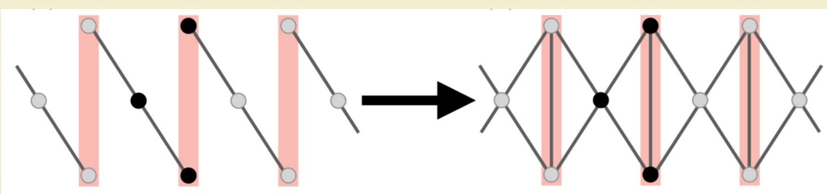
Many-body flatband localization

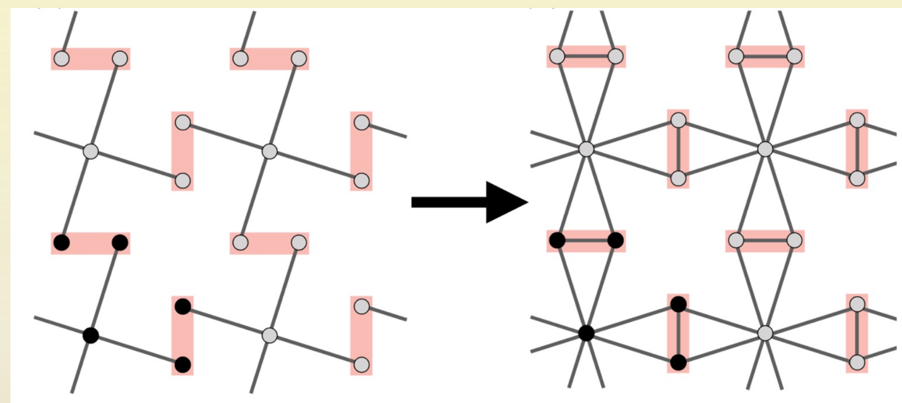
Carlo Danieli,[1] Alexei Andreanov,[2,3] and Sergej Flach[2,3,4]

All Bands Flat + Interactions ➔ Many Body Flat Band Localization

$\hat{\mathcal{H}}_{sp}/\hat{\mathcal{H}}_{int}$	SD	FD
SD		MBFBL
FD	MBFBL	MBFBL







No charge transport

PHYSICAL REVIEW B **102**, 041116(R) (2020)

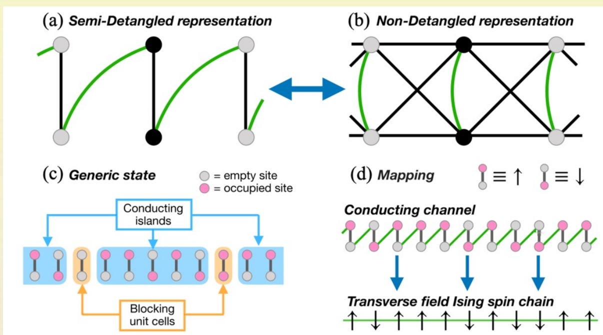
Rapid Communications

Many-body flatband localization

Carlo Danieli,[1] Alexei Andreanov,[2,3] and Sergej Flach[2,3,4]

All Bands Flat + Interactions ➔ Many Body Flat Band Localization

$\hat{\mathcal{H}}_{sp}/\hat{\mathcal{H}}_{int}$	SD	FD
SD		MBFBL
FD	MBFBL	MBFBL



Energy/heat transport?

blocking unit cell: full or empty i.e. only one state, blocks heat

d=1 : no heat transport except on Hilbert state subset of measure zero

d=2,3 : percolation transition upon varying filling factor,
even in percolating phase suppression of heat transport
due to charge disorder

PHYSICAL REVIEW B **104**, 144207 (2021)

Heat percolation in many-body flat-band localizing systems

Ihor Vakulchyk,[1,2,*] Carlo Danieli,[3,†] Alexei Andreanov,[1,2,‡] and Sergej Flach[1,2,§]

Not All Bands Flat + Interactions ➔ Disorder Free MBL !

Motivated by numerical indications of Dahm et al ➔

S. Tilleke, M. Daumann, and T. Dahm, Nearest neighbour particle-particle interaction in fermionic quasi one-dimensional flat band lattices, Z. Naturforsch. A **75**, 393 (2020).

M. Daumann, R. Steinigeweg, and T. Dahm, Many-body localization in translational invariant diamond ladders with flat bands, arXiv:2009.09705.

take a mixed dispersive + FB band structure

flat band : orthonormal

finite number of local unitaries detangle the FB ➔

Detangling flat bands into Fano lattices

Sergej Flach, Daniel Leykam, Joshua D. Bodyfelt, Peter Matthies and Anton S. Desyatnikov

EPL, **105** (2014) 30001

Add interactions in detangled basis with only density terms on the FB basis

Apply inverse unitaries and 'entangle'

Charges in FB are locked

Interaction between charges in dispersive states

Interaction induced scattering between dispersive states and locked FB charges

➔ THIS IS DISORDER FREE MBL !

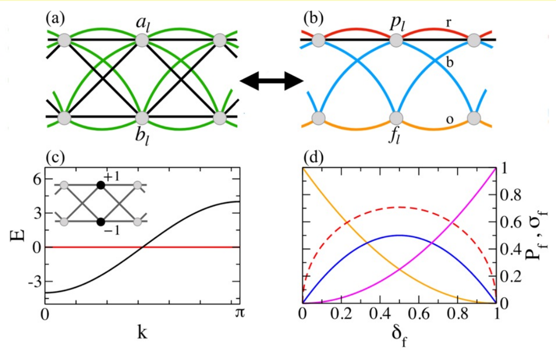
PHYSICAL REVIEW B **105**, L041113 (2022)

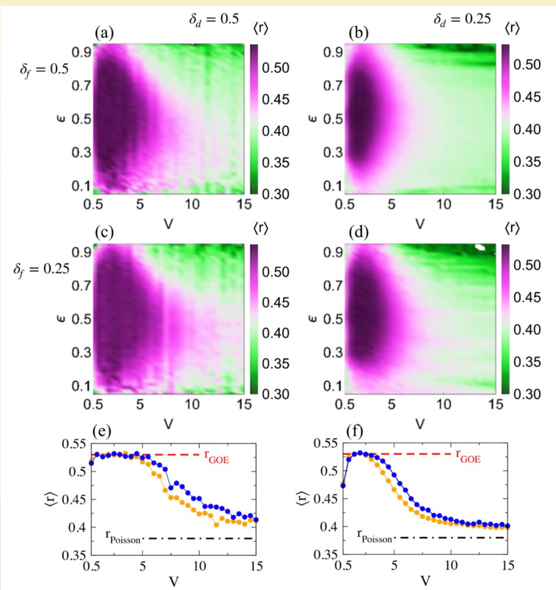
Letter

Many-body localization transition from flat-band fine tuning

Carlo Danieli,[1] Alexei Andreanov,[2,3] and Sergej Flach[2,3]

Not All Bands Flat + Interactions ➔ Many Body Flat Band Localization





$$\hat{\mathcal{H}}_{\mathrm{sp}} = \sum_l -[(\hat{a}_l^\dagger + \hat{b}_l^\dagger)(\hat{a}_{l+1} + \hat{b}_{l+1}) + \mathrm{H.c.}],$$

$$\hat{\mathcal{H}}_{\mathrm{int}} = \sum_l [\hat{n}_{a,l} + \hat{n}_{b,l}][\hat{n}_{a,l+1} + \hat{n}_{b,l+1}].$$

$$\hat{a}_l = (\hat{p}_l + \hat{f}_l)/\sqrt{2}$$

$$\hat{b}_l = (\hat{p}_l - \hat{f}_l)/\sqrt{2}$$

$$\hat{\mathcal{H}}_{\mathrm{sp}} = \hat{\mathcal{H}}_{\mathrm{sp}}^{\mathrm{f}} + \hat{\mathcal{H}}_{\mathrm{sp}}^{\mathrm{d}} \qquad \hat{\mathcal{H}}_{\mathrm{sp}}^{\mathrm{f}} = 0$$

$$\hat{\mathcal{H}}_{\mathrm{sp}}^{\mathrm{d}} = -2\sum\nolimits_l [\hat{p}_l^\dagger \hat{p}_{l+1} + \mathrm{H.c.}]$$

$$\hat{\mathcal{H}}^{\mathbf{q}} = \sum_l [\hat{\varepsilon}_l \hat{n}_{p,l} - 2(\hat{p}_l^\dagger \hat{p}_{l+1} + \mathrm{H.c.}) + V\hat{n}_{p,l}\hat{n}_{p,l+1}],$$

$$\hat{\varepsilon}_l = V(\hat{q}_{l+1} + \hat{q}_{l-1}), \qquad \hat{q}_l = \hat{f}_l^\dagger \hat{f}_l$$

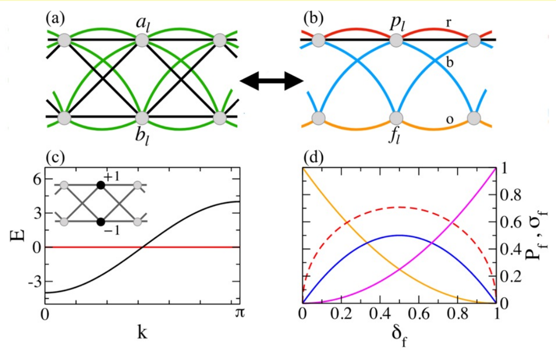
PHYSICAL REVIEW B **105**, L041113 (2022)

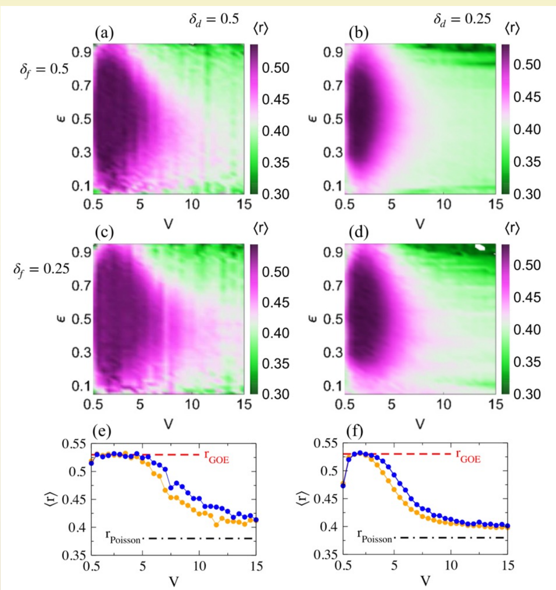
Letter

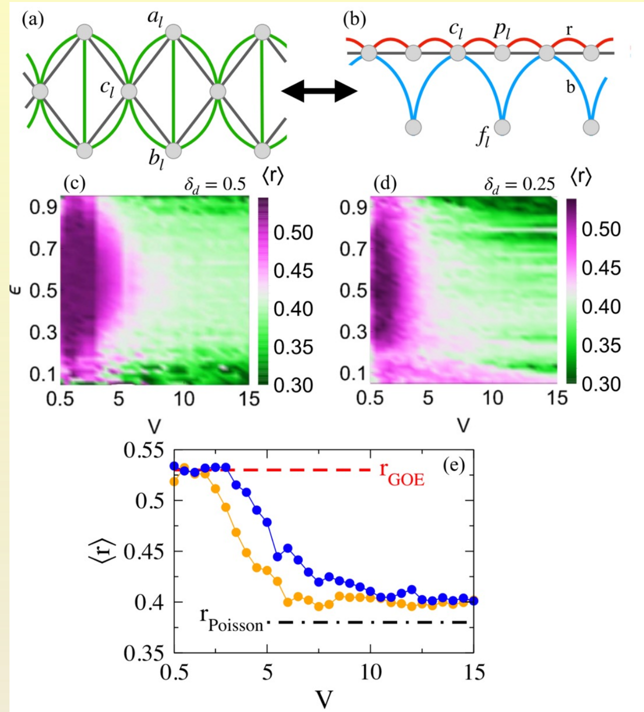
Many-body localization transition from flat-band fine tuning

Carlo Danieli,[1] Alexei Andreanov,[2,3] and Sergej Flach[2,3]

Not All Bands Flat + Interactions ➔ Many Body Flat Band Localization







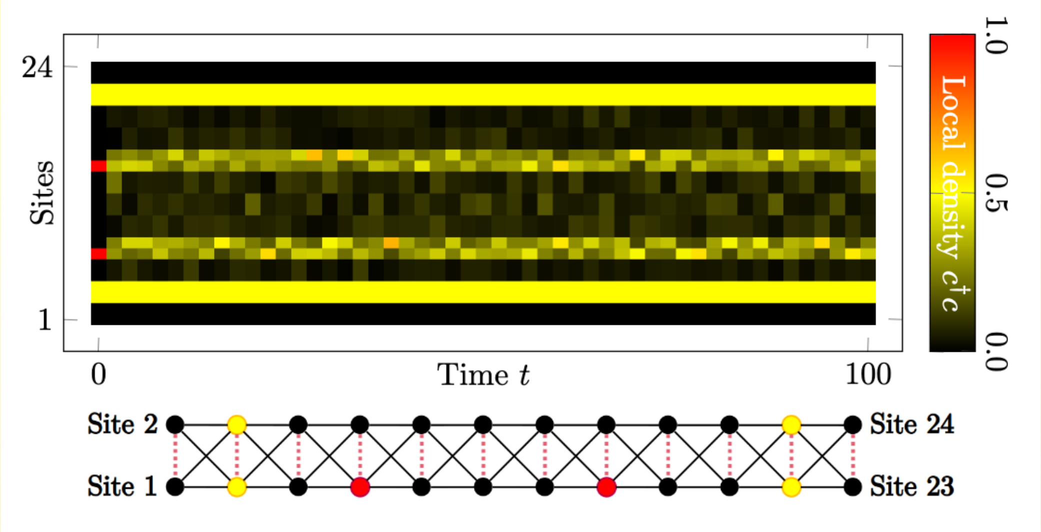
PHYSICAL REVIEW B **105**, L041113 (2022)

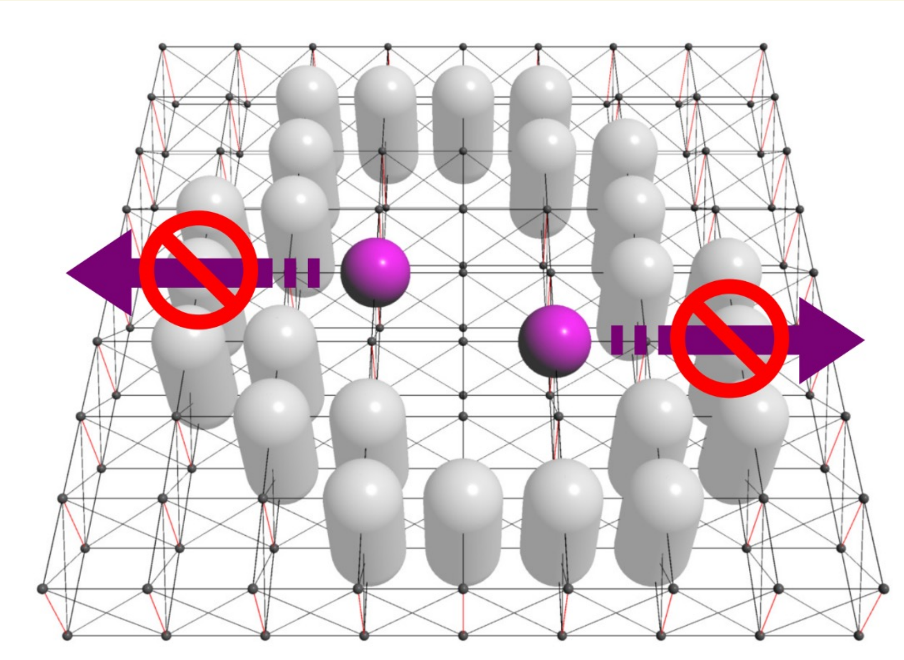
Letter

Many-body localization transition from flat-band fine tuning

Carlo Danieli,[1] Alexei Andreanov,[2,3] and Sergej Flach[2,3]

CLS can trap hard core bosons





Take homes

- Flat bands: macroscopic degeneracy
- Destructive interference produces compact localization: CLS
- CLS based FB generators systematically unravel all FB systems
- Finetuned FB model classes have rich internal structure
- Weak disorder: MITs and fractality edges
- Finetuned interactions: compact charge localization
  compact heat localization
  heat percolation
  disorder free MBL

People:

Alexei Andreanov
Carlo Danieli
Wulayimu Maimaiti
Hee Chul Park
Ajith Ramachandran
Daniel Leykam
Joshua Bodyfelt
Anton Desyatnikov
Oleg Gendelman
Ramaz Khomeriki
Daniel Leykam
Xiaoquan Yu
Ihor Vakulchyk
Mikhail Fistul
Yaroslav Zolotaryuk
Kun Woo Kim
Andrey Kolovsky
Alexandra Maluckov
Zhigang Chen (+ …)
Meng Sun
Ivan Savenko
Yuri Rubo
Y. D. Chong
G. Gligoric
Lj. Hadzievski
B. Malomed
Arindam Mallick
Nana Chang
Yeongjung Kim
Sanghoon Lee
Tilen Cadez

Reads:

Photonic Flat Bands
Daniel Leykam, SF
APL PHOTONICS 3, 070901 (2018)

Artificial flat band systems:
from lattice models to experiments
D. Leykam, A. Andreanov, SF
Adv. Phys.: X 3, 1473052 (2018)

http://pcs.ibs.re.kr **Publications**

Disclaimer: apologies for not citing you, them and that. There was not enough space-time.

Projectors

on one state $|\psi\rangle$: $P_\psi = |\psi\rangle\langle\psi|$

on many states : $P = \sum_n |\psi_n\rangle\langle\psi_n|$

on one band : $P_\mu = \sum_k |\psi_{k,\mu}\rangle\langle\psi_{k,\mu}|$

P_μ in real space basis

can be :
- compact localized
- exponentially localized
- algebraically localized (power law)

one band $\nu = 1$: P_ν compact

$\nu > 1$: $|\psi_{k\mu}\rangle$ k - independent : P_μ compact

$|\psi_{k\mu}\rangle$ k - dependent : P_μ exponentially localized (usually)