# Cavity magnonics

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- Intro: Spin waves and magnons
- Magnet-cavity coupling
- Level attraction
- Chiral magnon propagation
- Quantum magnonics

# Cavity magnonics

## Delft University of Technology

- Sanchar Sharma
- Xiang Zhang
- Tao Yu
- Marios Kounalakis

- Artem Bondarenko
- Enes Ilbuga
- YMB

- Toeno van der Sar and van der Sar Lab
- Slava Dobrovitski

## Tohoku University

- Mehrdad Elyasi
- Gerrit Bauer

## Cambridge University

- James Haigh

## Beijing Normal University

- Chuanpu Liu
- Haiming Yu

## University of Manitoba

- Bimu Yao
- Can-Min Hu

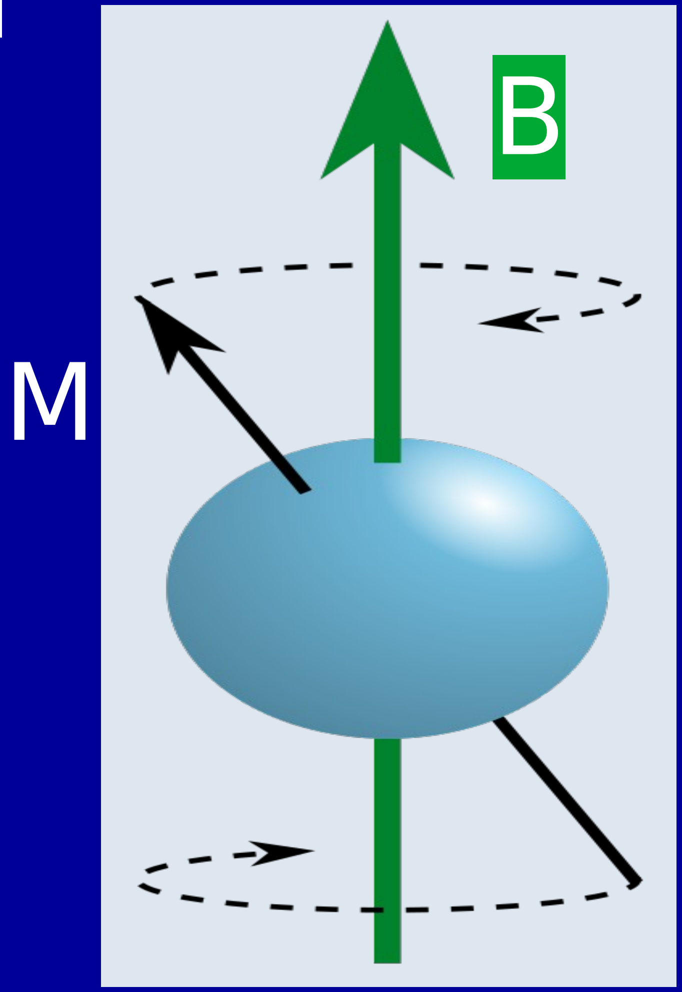
## Iran University of Science and Technology

- Babak Zare Rameshti

# Larmor precession

Magnetic moment in external field

Torque: $\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}$



# Magnons

**Magnons** are elementary excitations of magnetic structure

Classical limit (large occupation numbers): spin waves

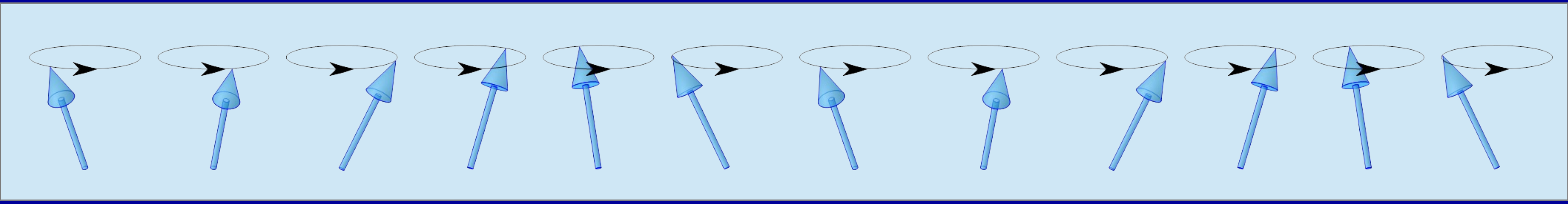


Image credit: Jens Böning, Wikimedia Commons

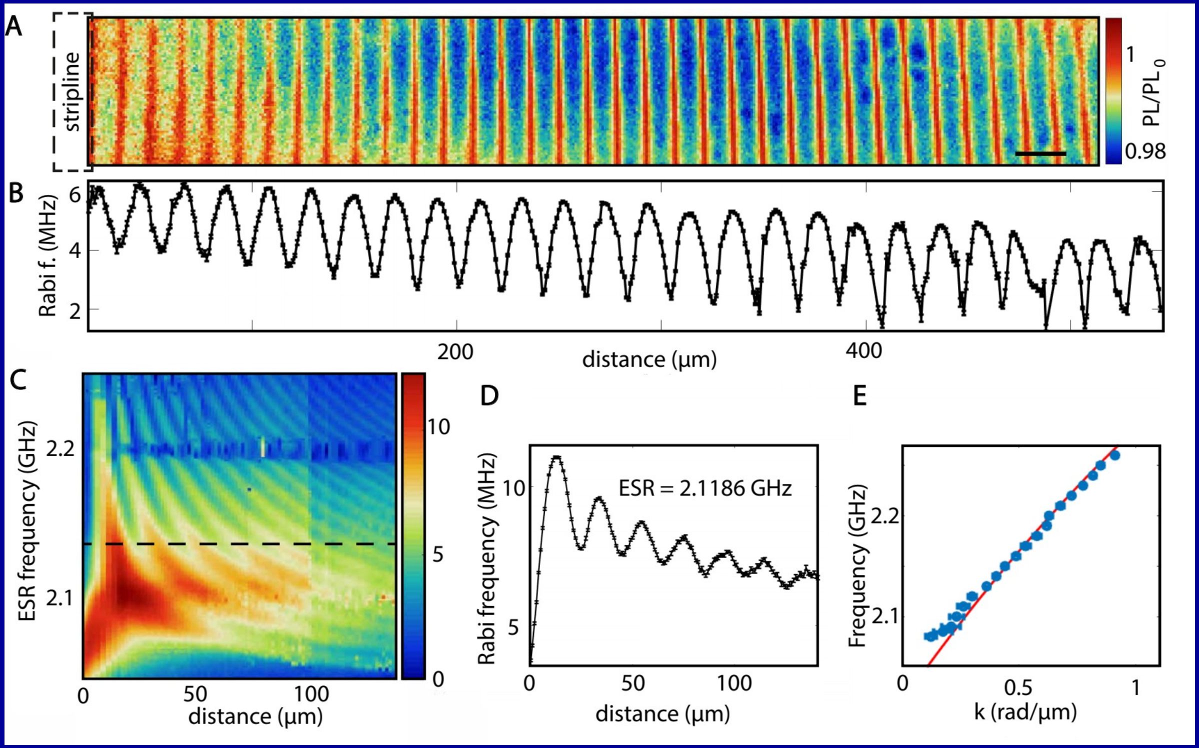
Spin Hamiltonian (simplest): $\hat{H}_S = -J\sum_{\langle ij\rangle}\hat{S}_i\cdot\hat{S}_j - g\mu_B B\cdot\sum_i \hat{S}_i$

Spin wave spectrum of an isotropic 1D FM chain:

$$\hbar\omega(k) = 2JS\,(1-\cos ka) + g\mu_B B$$

# Imaging of spin waves

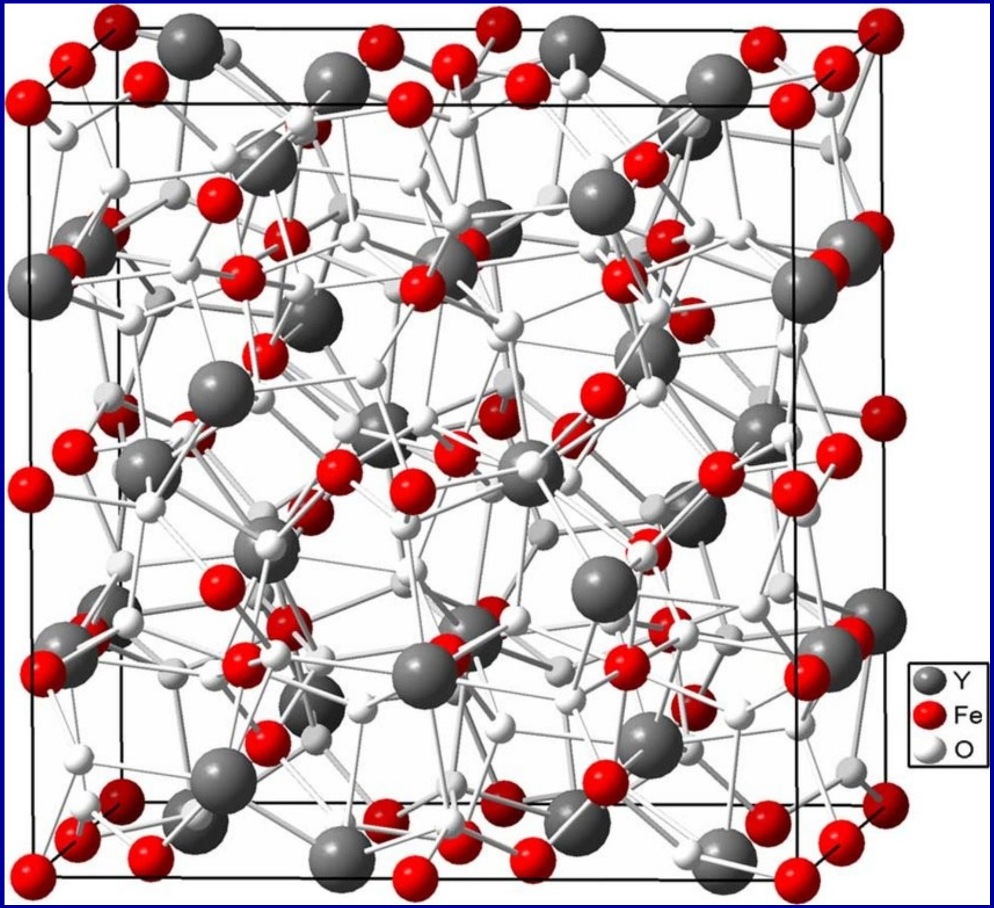
Spin waves in YIG films imaged by NV center magnetometry




I. Bertelli, J. J. Carmiggelt, T. Yu, B. G. Simon, C. C. Pothoven, G. E. W. Bauer, YMB, J. Aarts, and T. van der Sar, Science Adv.**6**, eabd3556 (2020).

# YIG

Yttrium Iron Garnet – ferrimagnetic insulator with the highest magnetic quality




Gilbert damping parameter:

$$\alpha \sim 10^{-4} - 10^{-5}$$

V. G. Harris et al
J. Magn. Magn. Mat. **321**, 2035 (2009)

# V[TCNE]x

Very recently: also vanadium tetracyanoethylene

Gilbert damping comparable to YIG

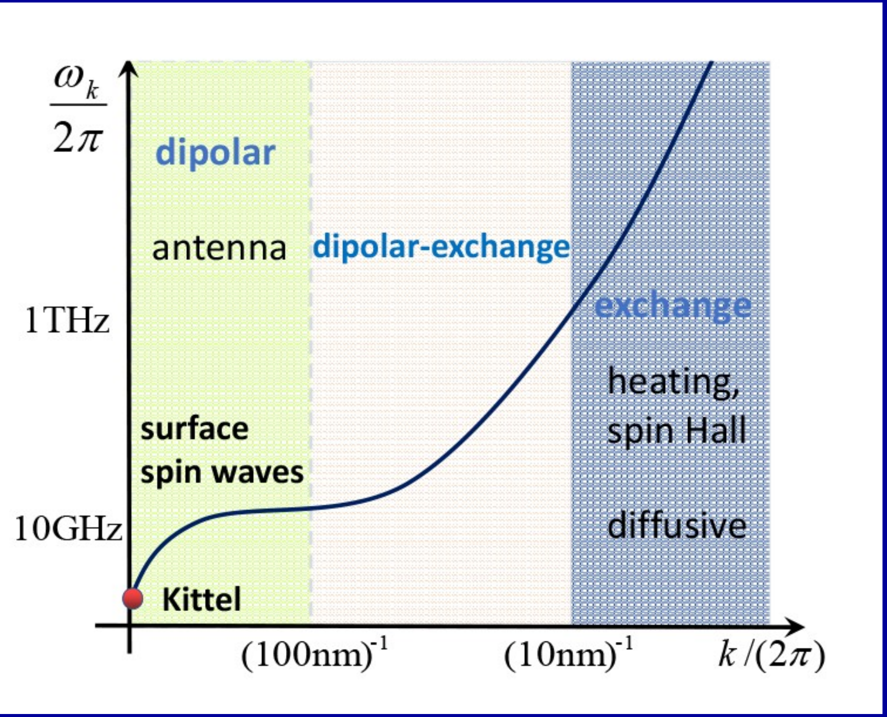
Q. Hu … G . Fuchs, arXiv:2212.04423

# Surface spin waves in YIG (Damon-Eschbach modes)

YIG: Dispersion relation is very anisotropic

Two competing mechanisms: Exchange and dipolar interactions

Spin waves travelling perpendicular to the magnetization



# Quantization of spin waves

Holstein-Primakoff transformation

$$\hat{S}_+ = \hbar\sqrt{2S}\sqrt{1-\frac{\hat{m}^\dagger\hat{m}}{2S}}\,\hat{m}$$

$$\hat{S}_- = \hbar\sqrt{2S}m^\dagger\sqrt{1-\frac{\hat{m}^\dagger\hat{m}}{2S}}$$

$$\hat{S}_z = \hbar\left(S-\hat{m}^\dagger\hat{m}\right)$$

Plane waves:

$$\hat{m}_k = \frac{1}{\sqrt{N}}\sum_{R_i}\exp\left(-ik\cdot R_i\right)\hat{m}_i$$

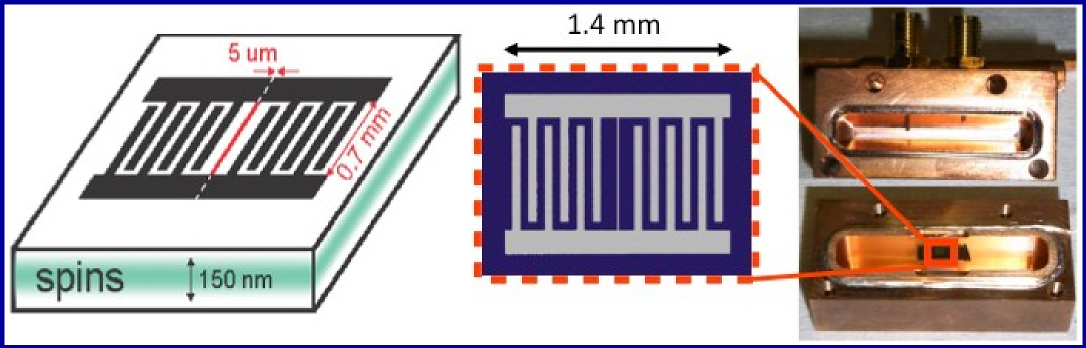
Linearized transformation:

$$\hat{m}^\dagger\hat{m} \ll S$$

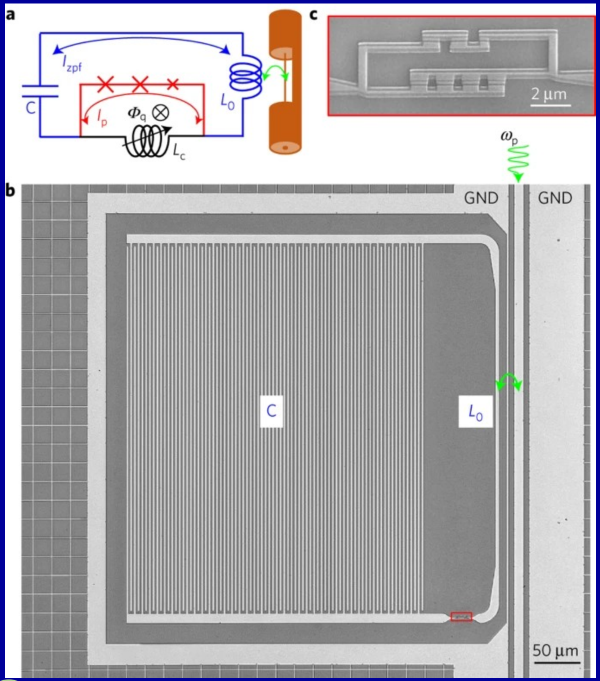
One-mode linear Hamiltonian:

$$\hat{H} = \hbar\omega_m\hat{m}^\dagger\hat{m}$$

# Microwave cavities




Copper box cavity: A. Bienfat .... P. Bertet, Nature **531** 74 (2016)




Lumped-element LC resonator: F. Yoshihara .... K. Semba, Nature Physics **13** 44 (2017)

# Microwave cavities

Generally: Multimode cavities

Quantize vector potential (homogeneous situation):

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}^+(\mathbf{r},t) + \mathbf{A}^-(\mathbf{r},t), \quad \mathbf{A}^+ = (\mathbf{A}^-)^\dagger$$

$$\mathbf{A}^+(\mathbf{r},t) = \sum_k u_k(\mathbf{r})\hat{a}_k e^{-i\omega_k t}, \omega_k = ck/n \qquad \left(\nabla^2 + k^2\right) u_k = 0$$

Free cavity Hamiltonian: $\hat{H} = \sum_k \hbar\omega_k a_k^\dagger \hat{a}_k$

Fields: $$\hat{\mathbf{E}}^+(\mathbf{r},t) = i\sum_k \sqrt{\frac{\hbar\omega_k}{2V\varepsilon\varepsilon_0}} u_k(\mathbf{r})\hat{a}_k e^{-i\omega_k t}$$

$$\hat{\mathbf{B}}^+(\mathbf{r},t) = i\sum_k \sqrt{\frac{\hbar}{2V\varepsilon\varepsilon_0\omega_k}} \nabla \times u_k(\mathbf{r})\hat{a}_k e^{-i\omega_k t}$$

Mode volume: $V_k = \dfrac{\int |E_k|^2 dV}{\max |E_k|^2}$

In most situations: Want to work with one mode

# Magnet-cavity Interaction

Mechanism: interaction of magnetization with the cavity field $M \cdot B$

Hamiltonian of an interaction of a single cavity mode with a single (almost resonant) magnon mode:

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{m}^\dagger \hat{m} + g\left(\hat{a}^\dagger \hat{m} + \hat{a}\hat{m}^\dagger\right)$$

Photon

Magnon

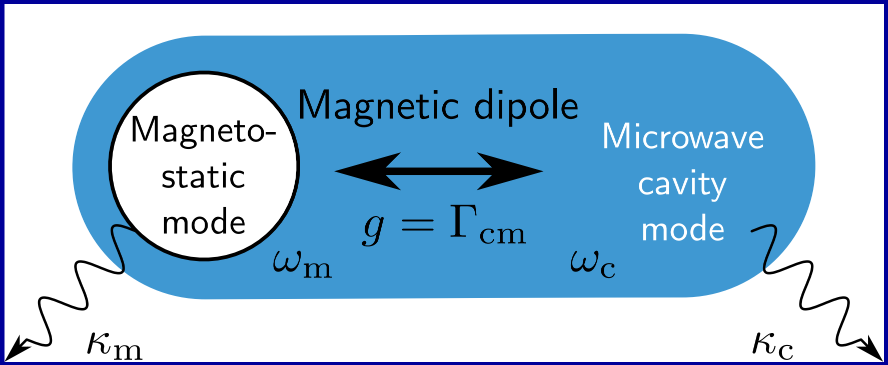
Interaction (RWA)

Strong coupling in a cavity predicted by Soykal and Flatte, Phys. Rev. Lett. **104**, 077202 (2010)

g – coupling constant

Strong coupling regime means:

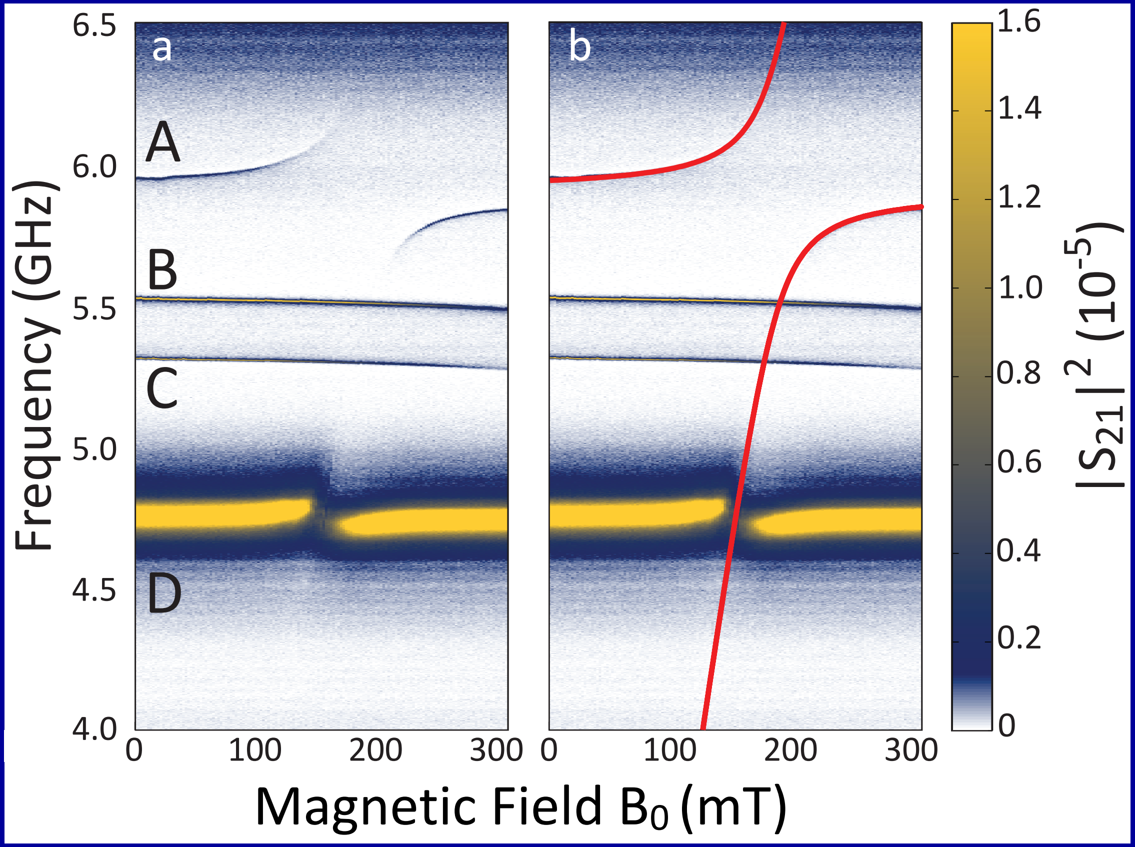
$$\kappa_c, \kappa_m \ll g \ll \omega_c, \omega_m$$



# Interaction

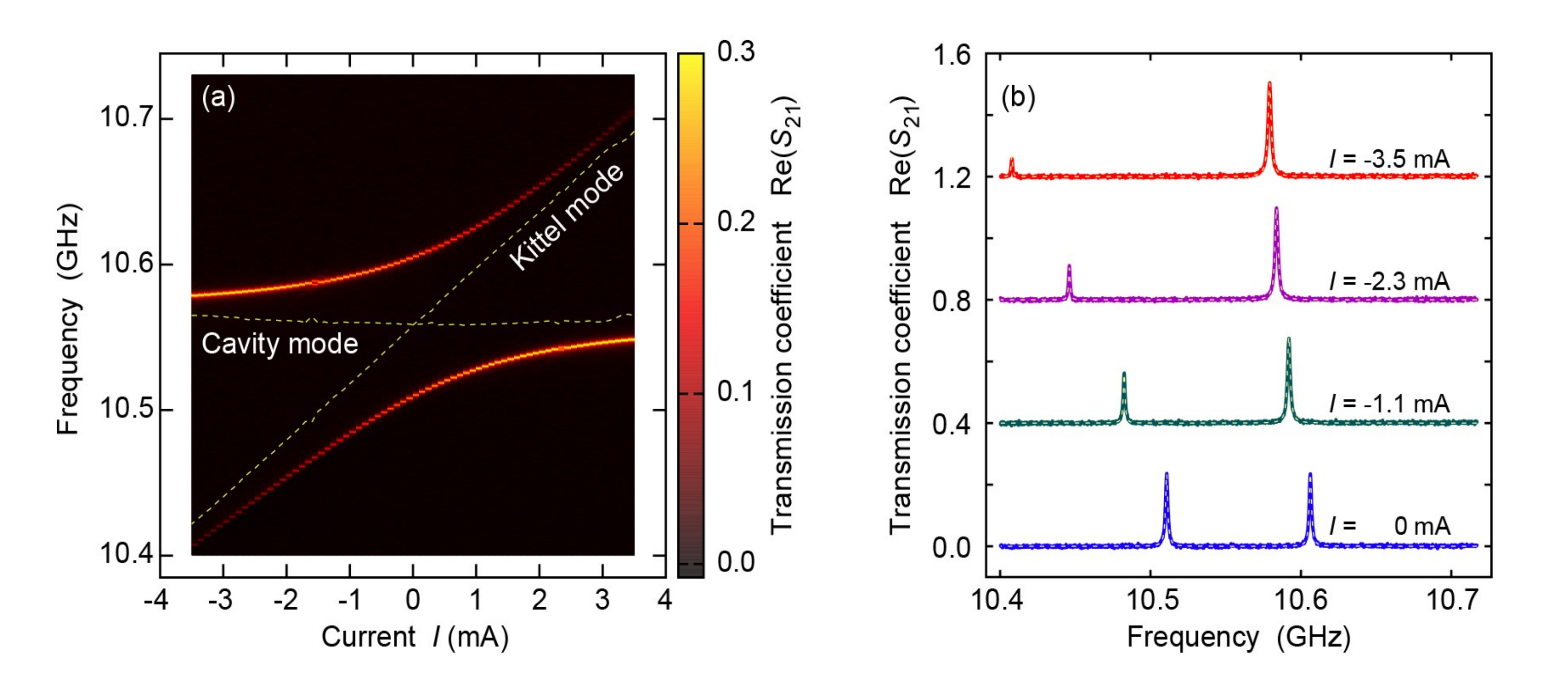
YIG film in a cavity

Hübl et al,
Phys. Rev. Lett. **111**, 127003 (2013)



# Interaction

Tabuchi et al,
Phys. Rev. Lett. **113**, 083603 (2014)




Normal mode splitting between a magnon (YIG sphere) and a cavity mode

# Magnon spintronics

Spin waves can carry information:

Ferromagnetic metals: Spin current is carried by electrons
– Ohmic dissipation

Ferromagnetic insulators: Spin current carried by spin waves
– Weak intrinsic damping of spin waves

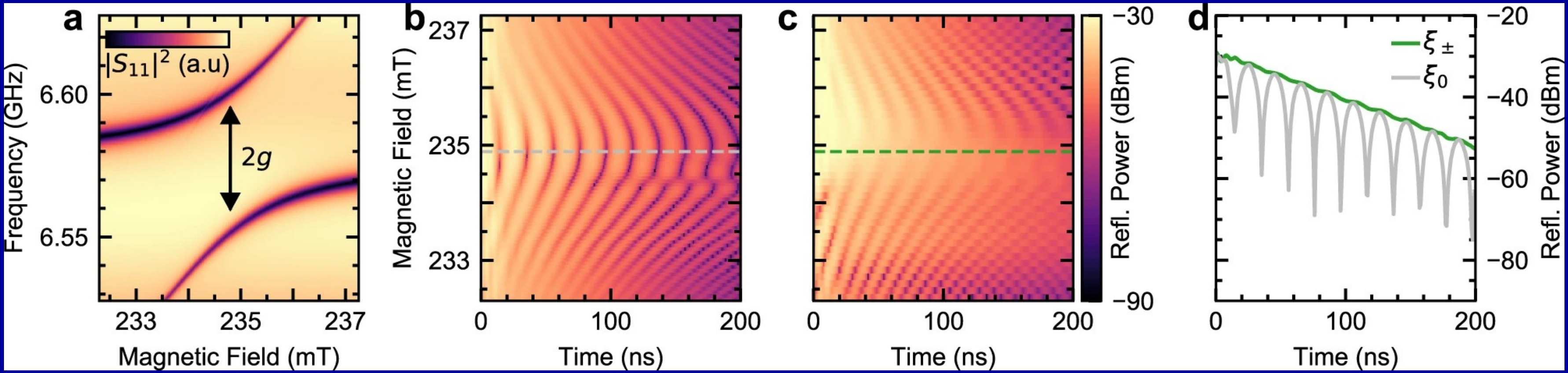
We want to be able to excite, manipulate, and read out spin waves

*Cavity Magnonics*, Babak Zare Rameshti, Silvia Viola Kusminskiy, James A. Haigh, Koji Usami, Dany Lachance-Quirion, Yasunobu Nakamura, Can-Ming Hu, Hong X. Tang, Gerrit E. W. Bauer, and YMB, Physics Reports **979**, 1 (2022).

*Quantum magnonics: When magnon spintronics meets quantum information science*, H. Y. Yuan, Yunshan Cao, Akashdeep Kamra, Rembert A. Duine, and Peng Yan, Physics Reports **965**, 1 (2022).

# Coherent manipulation

Normal mode splitting between a magnon (YIG sphere) and a cavity mode: Cavity-magnon polariton



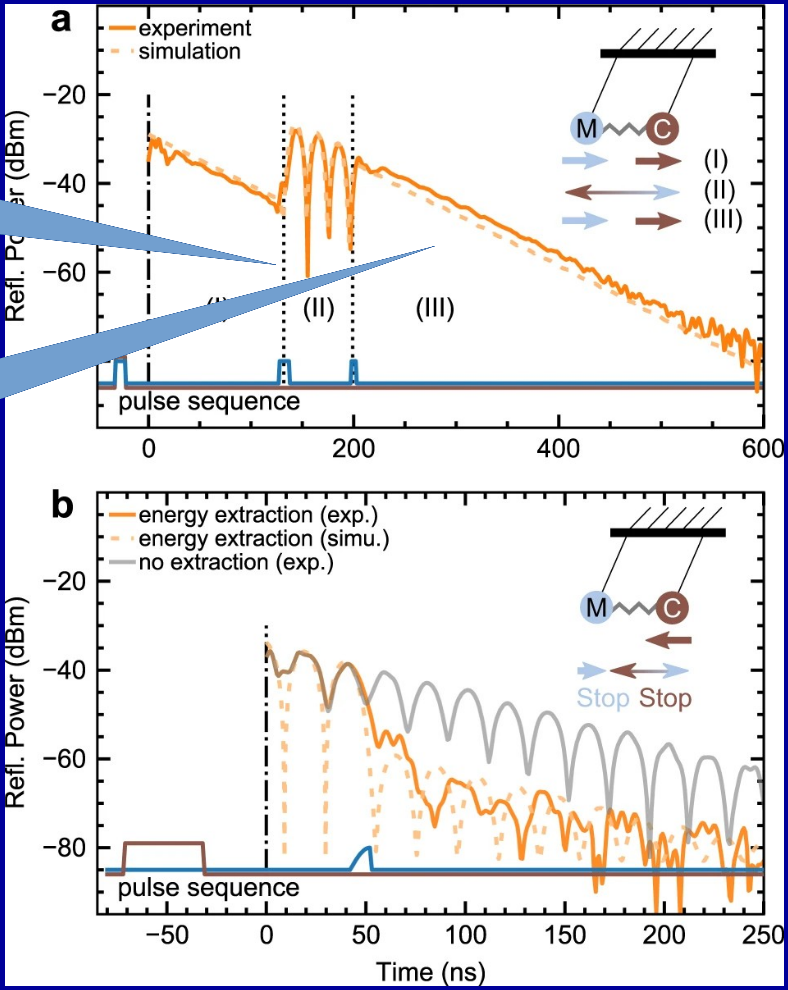

They can drive cavity and magnon modes independently → coherent superposition

Wolz et al, Communications Physics **3**, 3 (2020)

# Coherent manipulation

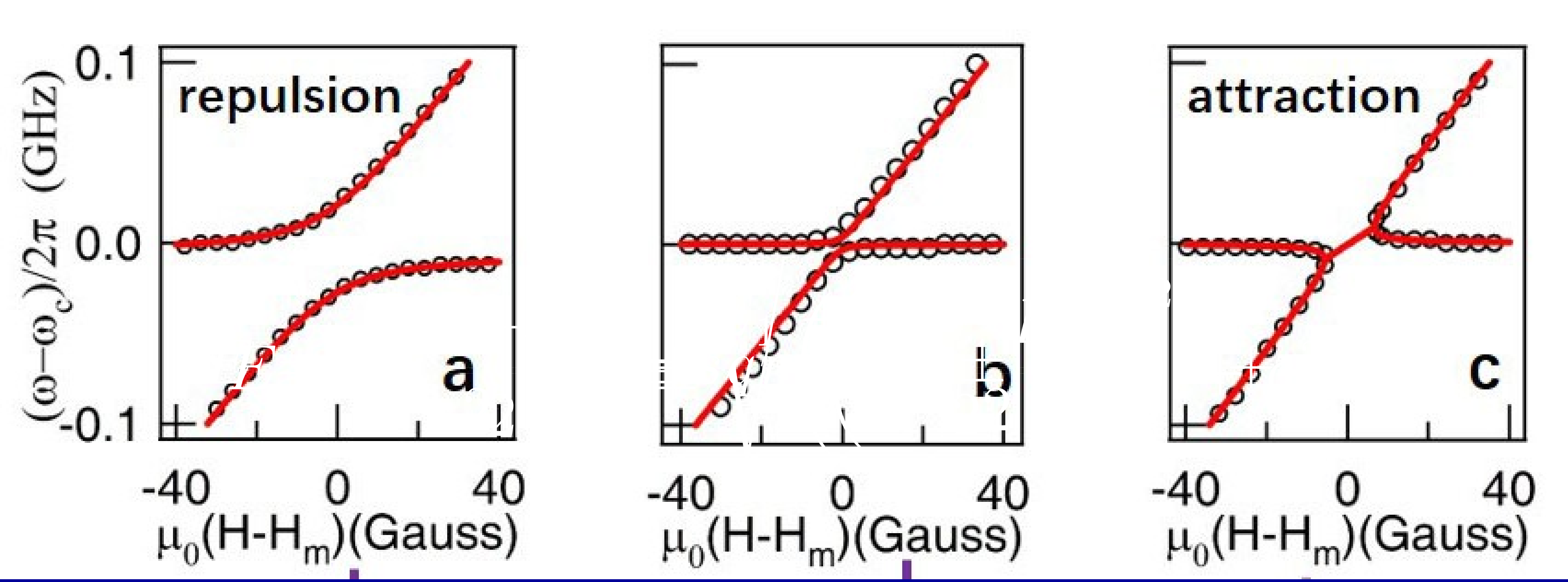
Here the pulse brings the system into coherent superposition: Rabi oscillations

No superposition, just one branch: Exponential decay




Wolz et al, Communications Physics **3**, 3 (2020)

# Level repulsion and attraction




$$E = \frac{E_1 + E_2}{2} \pm \sqrt{\left(\frac{E_1 - E_2}{2}\right)^2 + g^2}$$

# Dissipative coupling

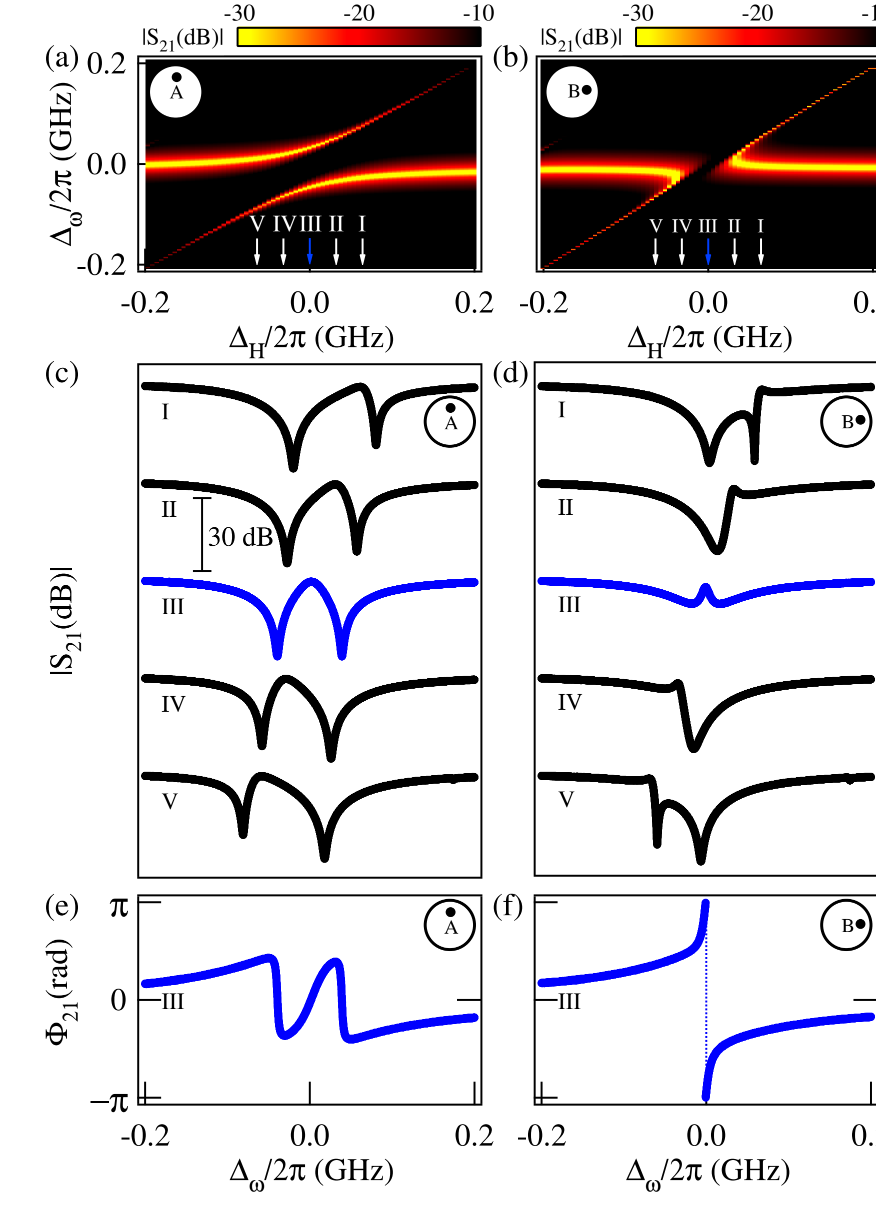
A magnetic sphere in a lossy Fabry-Perot cavity

Cavity photon spectrum:
- Localized waves at resonances
- Continuous travelling waves away from resonances

Coherent coupling to resonant modes – leads to level repulsion

Coupling to travelling modes – dissipative, leads to level attraction
- Non-Hermitian Hamiltonian
- Competition between level repulsion and attraction

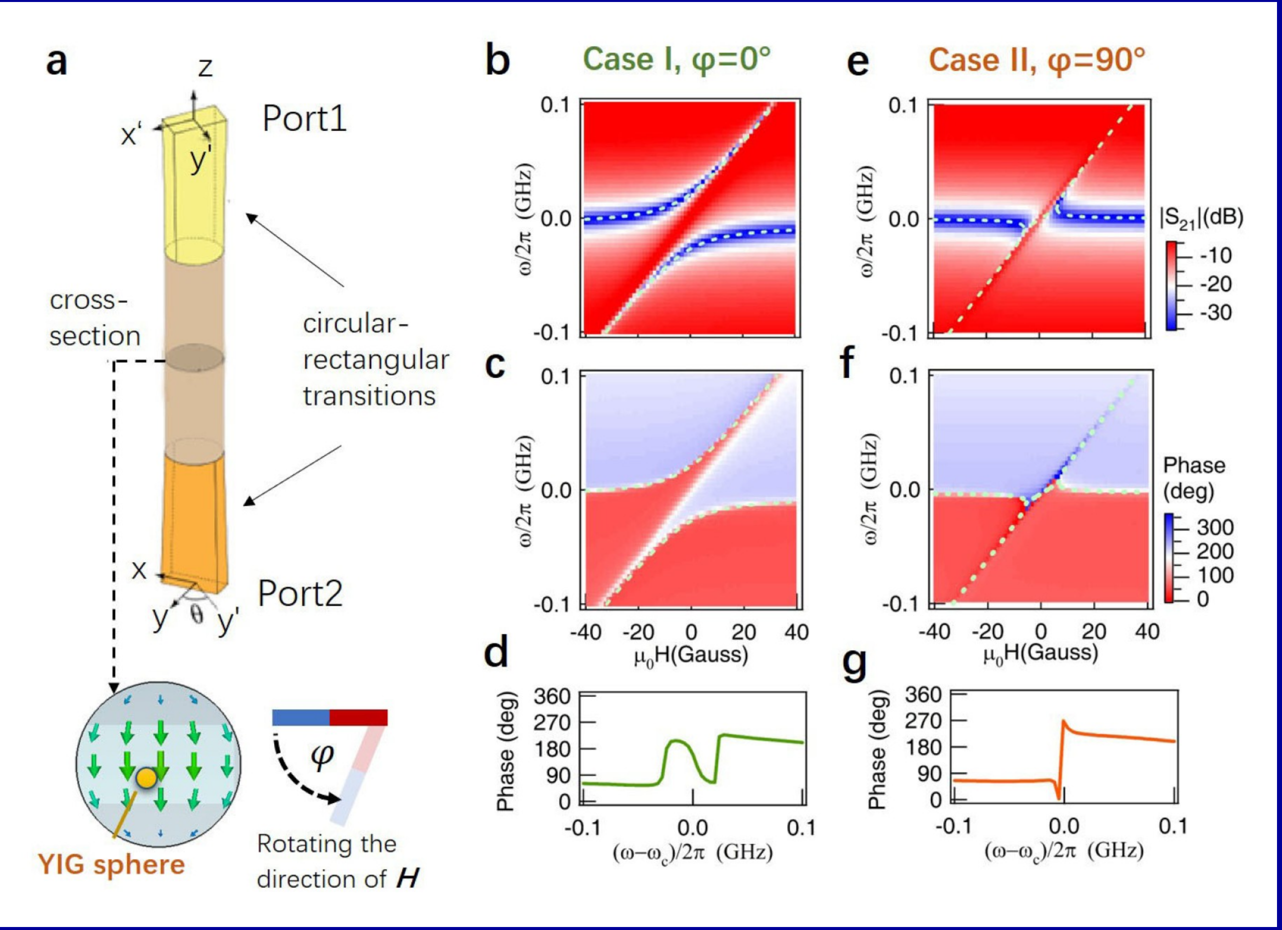
# Level attraction




M. Harder … C.-M. Hu, Phys. Rev. Lett. 121, 137203 (2018)

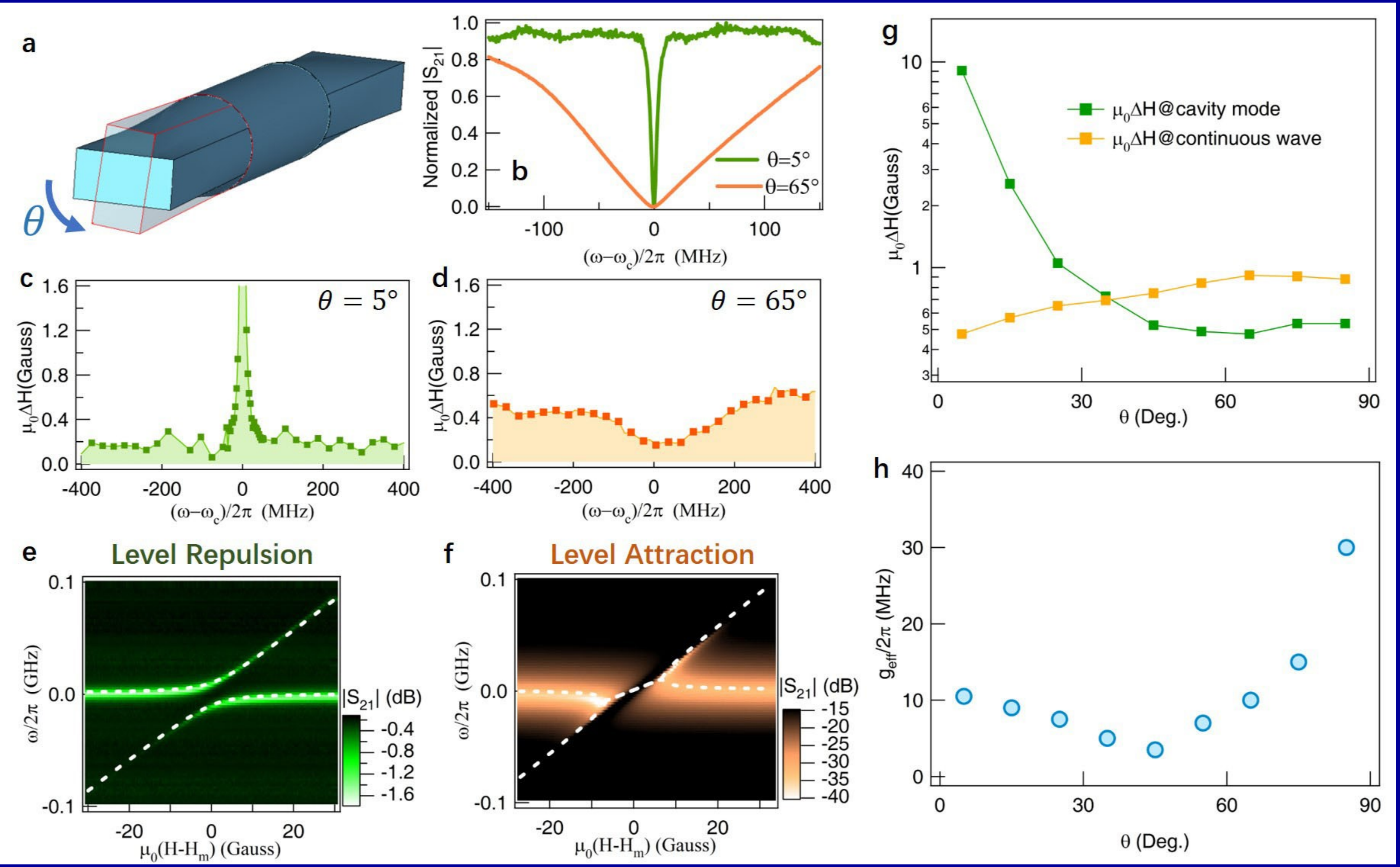
# Level attraction

A magnetic sphere in a lossy Fabry-Perot cavity




B. Yao, T. Yu, X. Zhang, W. Lu, Y. Gui, C.-M. Hu, and YMB, Phys. Rev. B **100,** 214426 (20

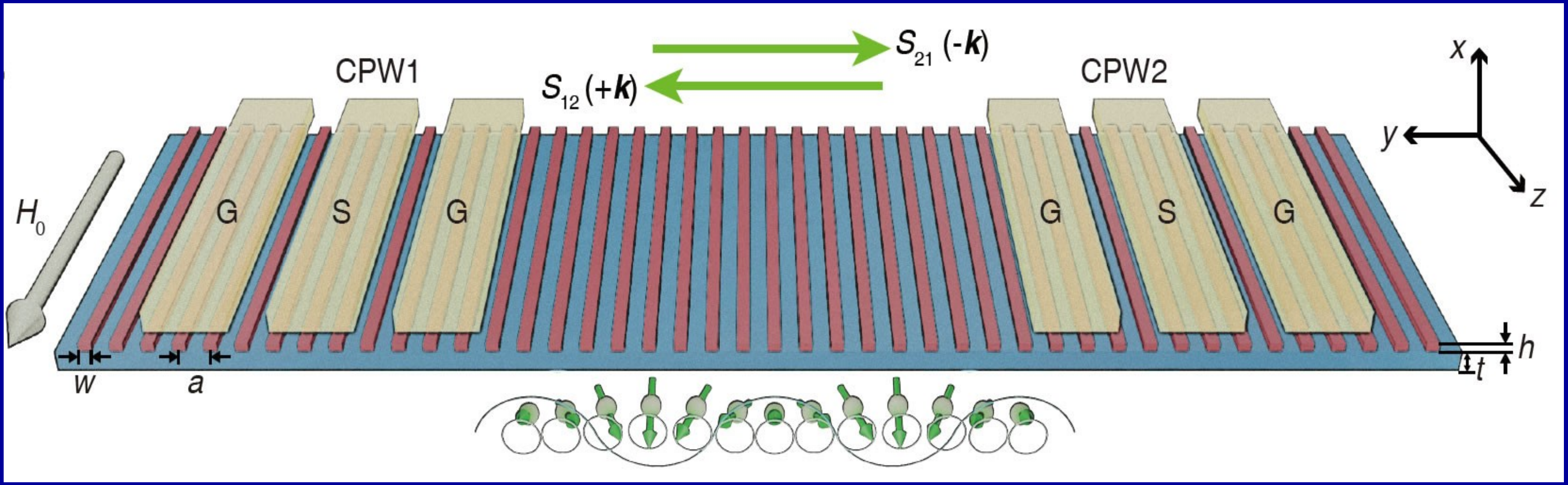
# Dissipative coupling




 Yaroslav M. Blanter ICTP Summer School, September 2023

# Experiment: Excitation of spin waves

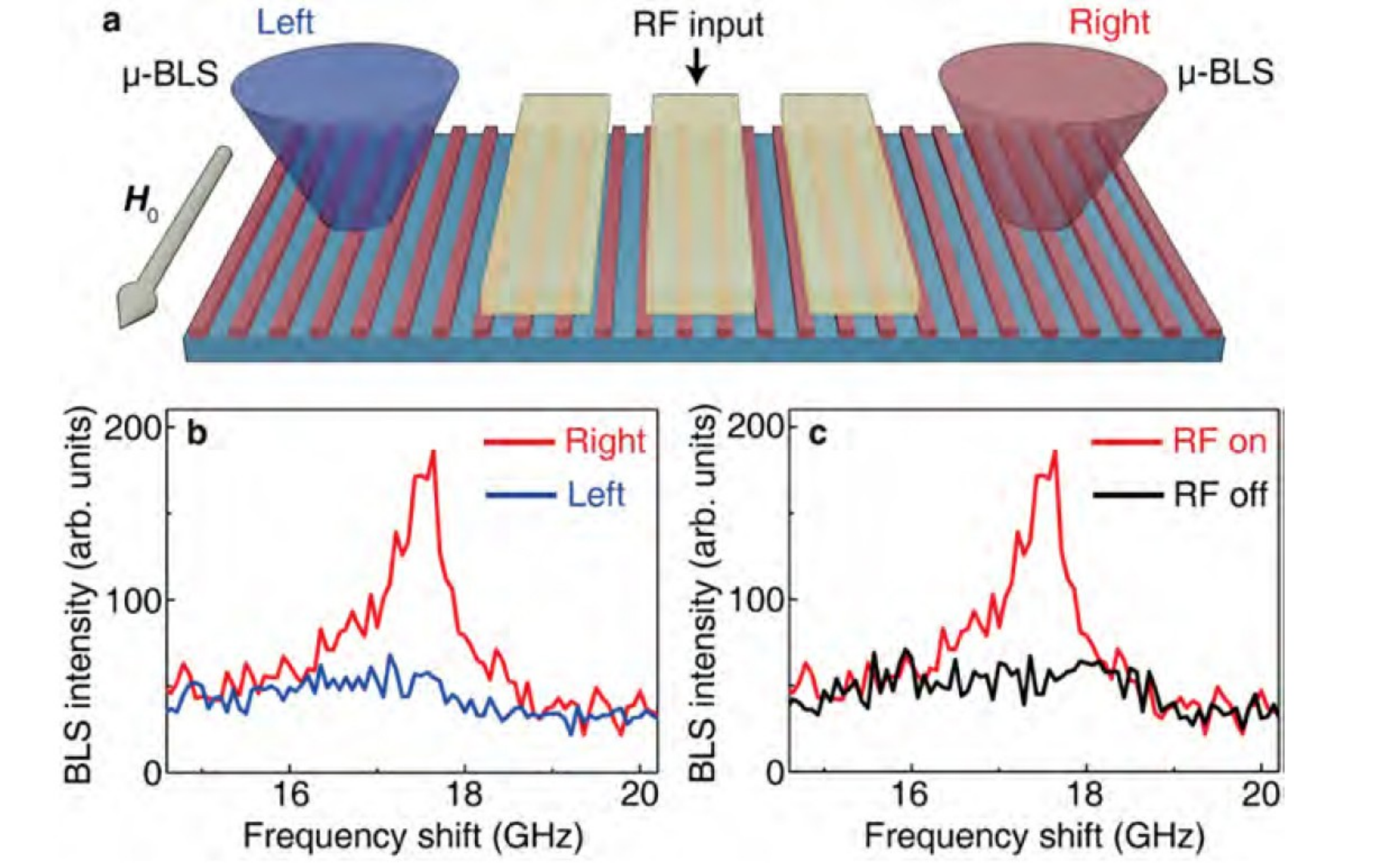
Magnetic nanowire array; FMR used to excite spin waves




Thickness: Not sufficient to support surface modes (20 nm)

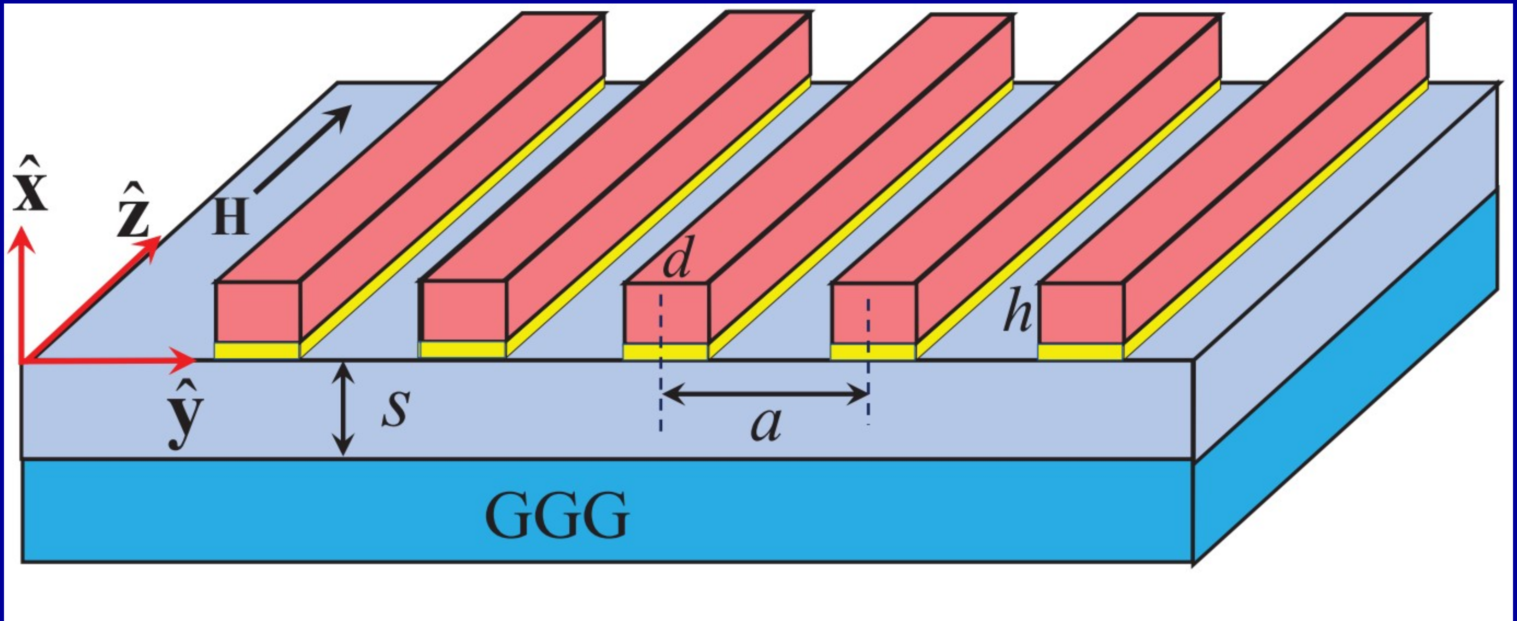
J. Chen, T. Yu, C. Liu, T. Liu, M. Madami, Ka Shen, J. Zhang, S. Tu, M. Shah Alam, Ke Xia, M. Wu, G. Gubbiotti, YMB, G E. W. Bauer, and H. Yu, Phys. Rev. B **100,** 104427 (2019)

# Experiment: BLS




J. Chen, T. Yu, C. Liu, T. Liu, M. Madami, Ka Shen, J. Zhang, S. Tu, M. Shah Alam, Ke Xia, M. Wu, G. Gubbiotti, YMB, G E. W. Bauer, and H. Yu, Phys. Rev. B **100,** 104427 (2019)

# Theoretical model




We take for calculations s = 20 nm, a =200 nm, d = 100 nm

Modes in the array: Close to the ferromagnetic resonance

$$\omega_{FMR} = \gamma \mu_0 \sqrt{H(H + M_0)}$$

Saturation magnatization

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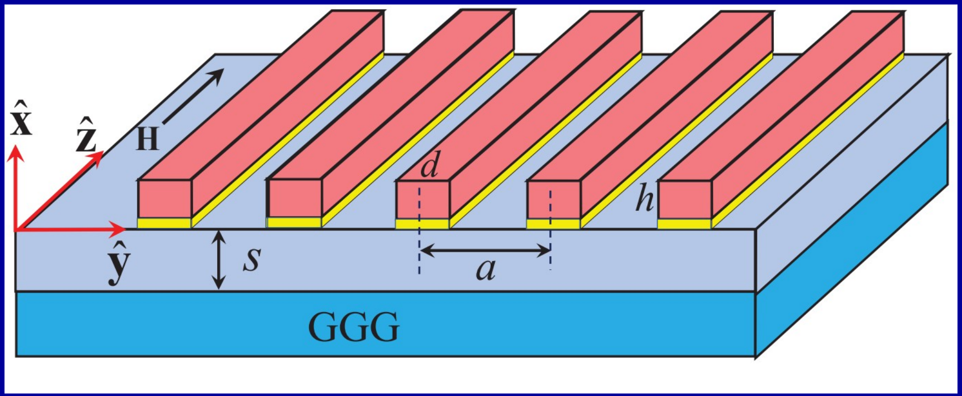
# Spin waves in films

Landau-Lifshits equation

$$\frac{dM}{dt} = -\gamma\mu_0 M \times (H + H_{dip} + H_{ex})$$

dipolar

exchange




Spin waves can travel in both directions

Spin-momentum locking: stray fields are polarized in a certain direction

$$B(k) \propto \left(m_z - i\frac{k}{|k|}m_y\right)$$

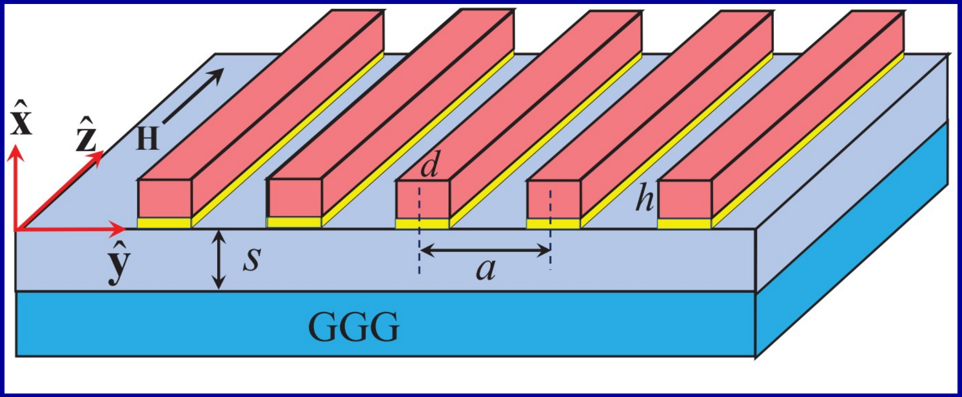
T. Yu, C. Liu, H. Yu, YMB, and G E. W. Bauer, Phys. Rev. B **99**.134424 (2019)

# Dipolar interactions

Free energy

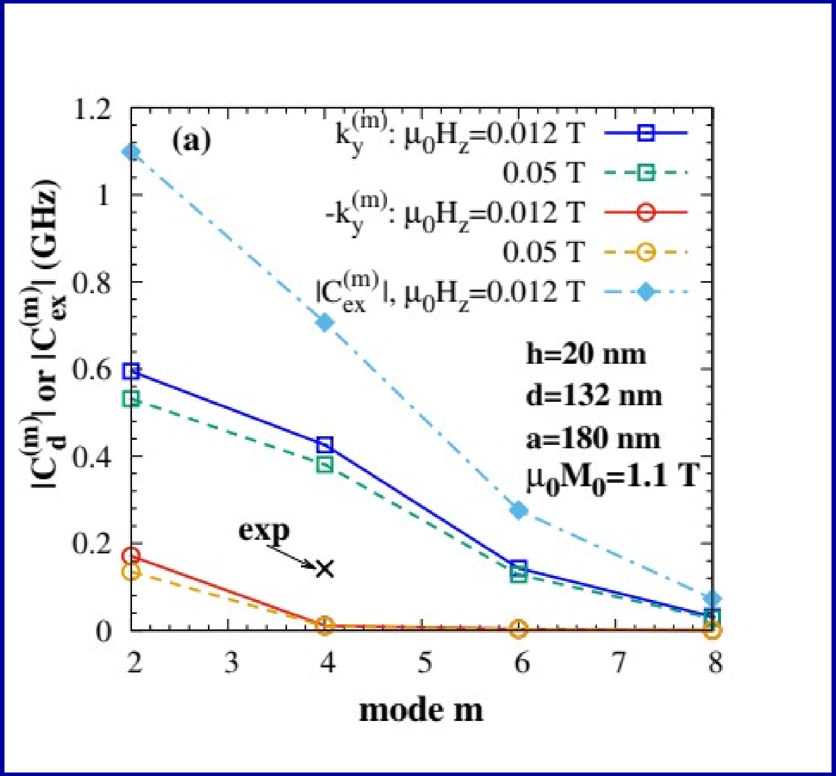
$$F = -\mu_0 \int M h^{dip} dr$$

$$h_\beta^{dip} = \frac{1}{4\pi} \partial_\beta \int dr' \frac{\partial_\alpha M_\alpha^{arr}(r')}{|r - r'|}$$

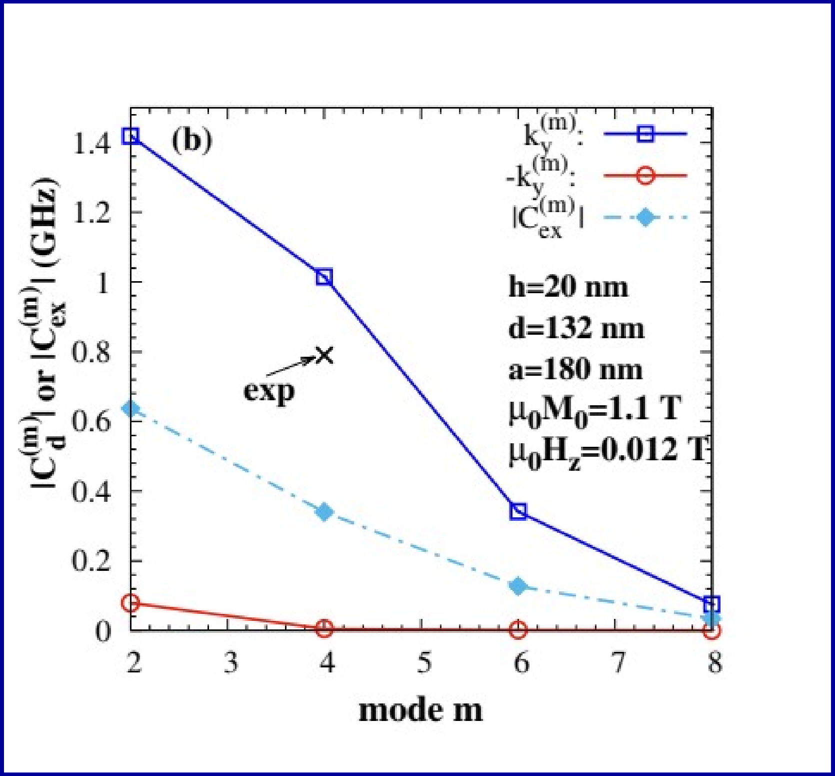



- Only spin waves travelling in one direction generate dipolar fields above the film and thus couple to the array
- If the spin wave in the film is circularly polarized, it can only be excited in one direction
- Ellipticity leads to reduction of chirality

# Couplings



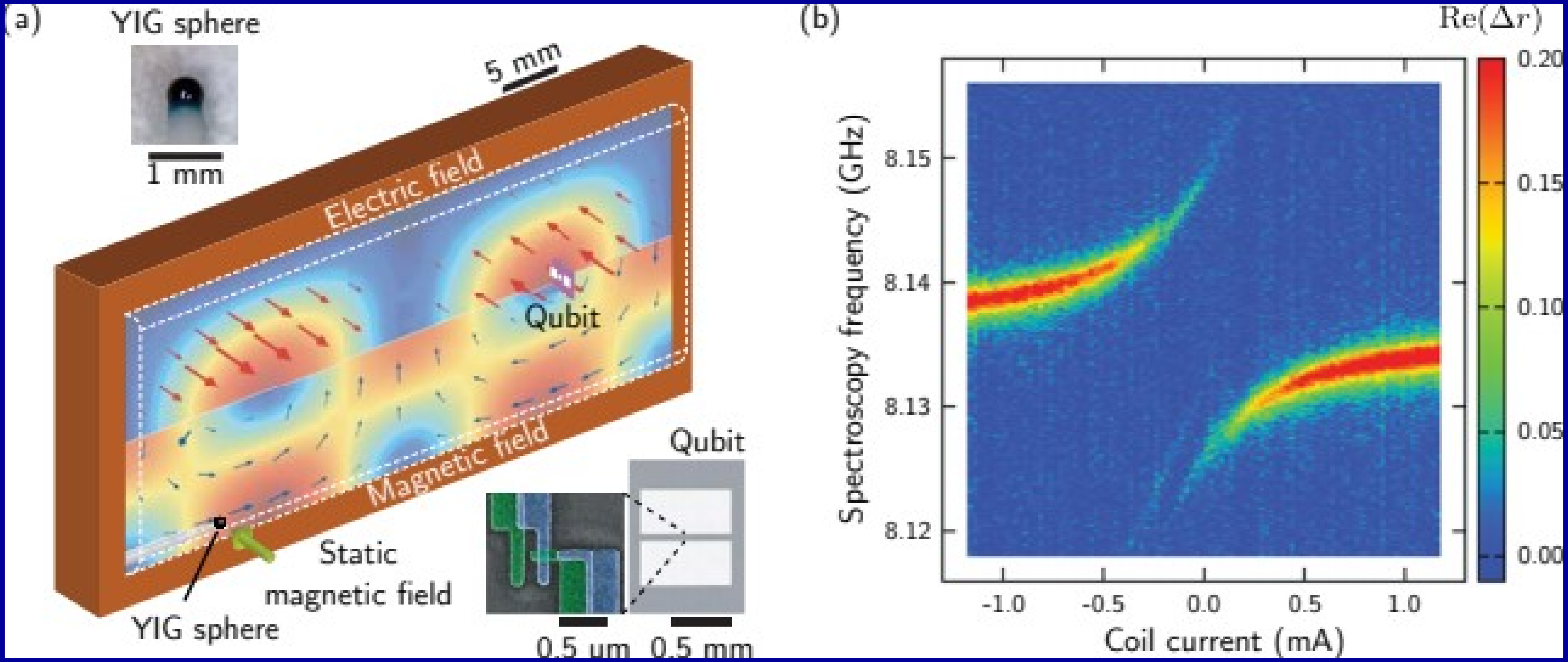

Parallel




Anti-parallel

# Coupling of a magnon to a qubit

Coherent coupling of a magnon to a qubit via a cavity

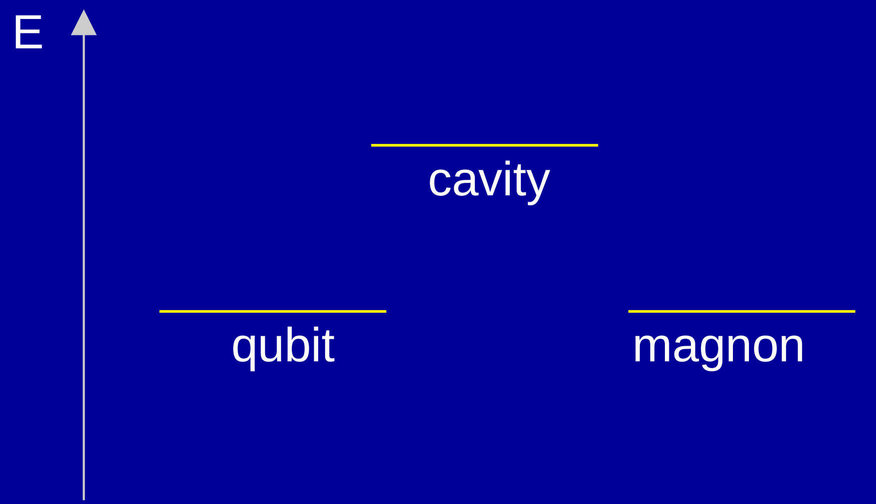



The qubit is resonant with the magnon; the cavity is detuned and only serves to provide controlled interaction

Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Science **349**, 405 (2015)

# Resonant coupling

Qubit-magon interaction: Via the 2nd order perturbation theory




$$g_{m-q} = \frac{g_{m-c}g_{c-q}}{\omega_c - \omega_m}$$

Strong coupling:

$$g_{q-m} \gg \kappa_m, \kappa_q$$

Effective Hamiltonian:

$$\hat{H} = \hat{H}_q + \hat{H}_m + \hat{H}_{int}$$

$$\hat{H}_q = \hbar\omega_q \hat{c}^\dagger \hat{c} + K \hat{c}^\dagger \hat{c}^\dagger \hat{c}\hat{c}$$

$$\hat{H}_m = \hbar\omega_m \hat{m}^\dagger \hat{m}$$

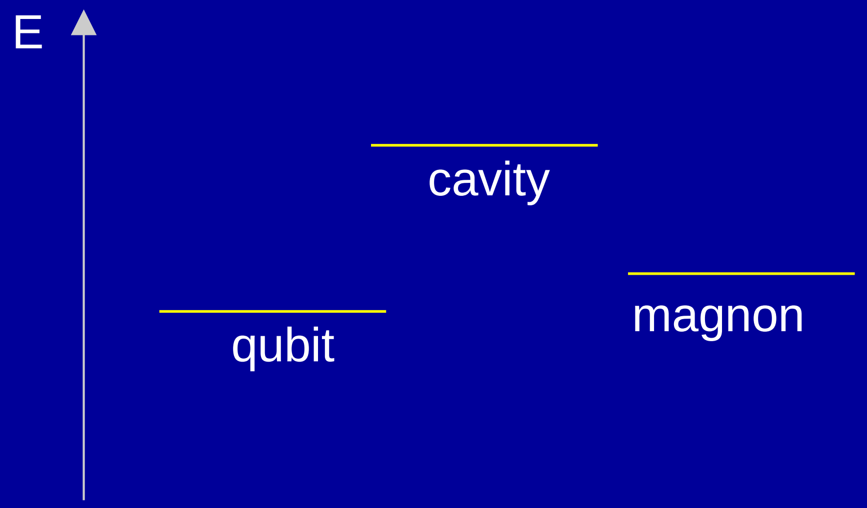
$$\hat{H}_{int} = g_{m-q}\left(\hat{m}^\dagger \hat{c} + \hat{m}\hat{c}^\dagger\right)$$

Same as circuit QED

# Dispersive coupling

Magnon and qubit are detuned

$$|\omega_q - \omega_m \gg \kappa_q, \kappa_m$$




Non-linear interaction (cross-Kerr): $\hat{H}_{int} = \tilde{K}\hat{m}^\dagger\hat{m}\hat{c}^\dagger\hat{c}$

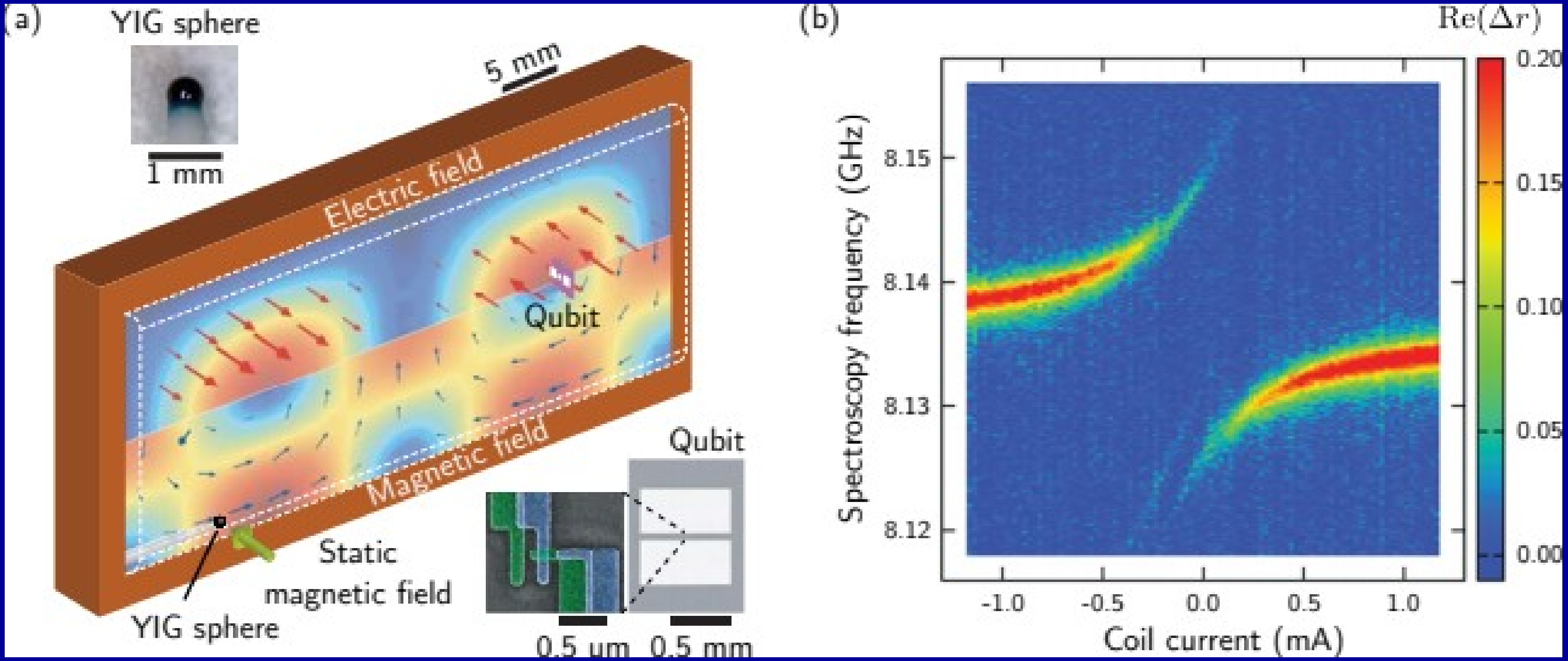
$$\hat{H}_q = \hbar\omega_q\hat{c}^\dagger\hat{c} + K\hat{c}^\dagger\hat{c}^\dagger\hat{c}\hat{c}$$

$$\hat{H}_m = \hbar\omega_m\hat{m}^\dagger\hat{m}$$

Shift of the frequency of one system
depending on the state of another system

# Resonant coupling

Coherent coupling of a magnon to a qubit via a cavity




Strong coupling between a qubit and a magnon

Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Science **349**, 405 (2015)

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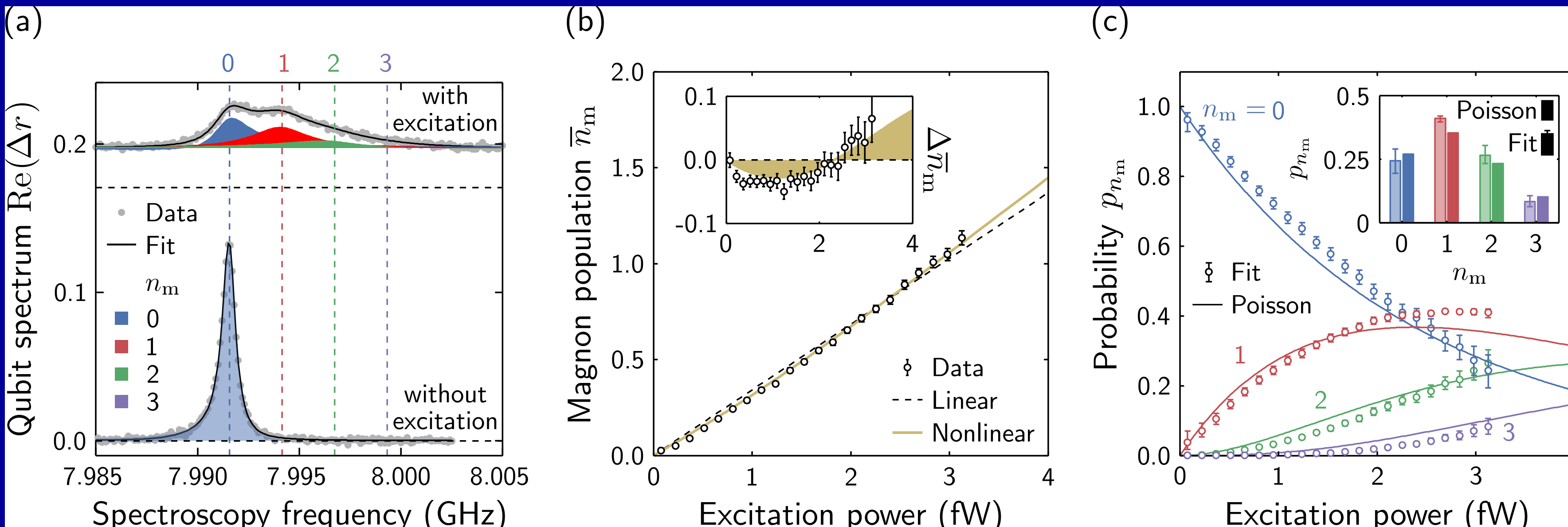
# Dispersive coupling

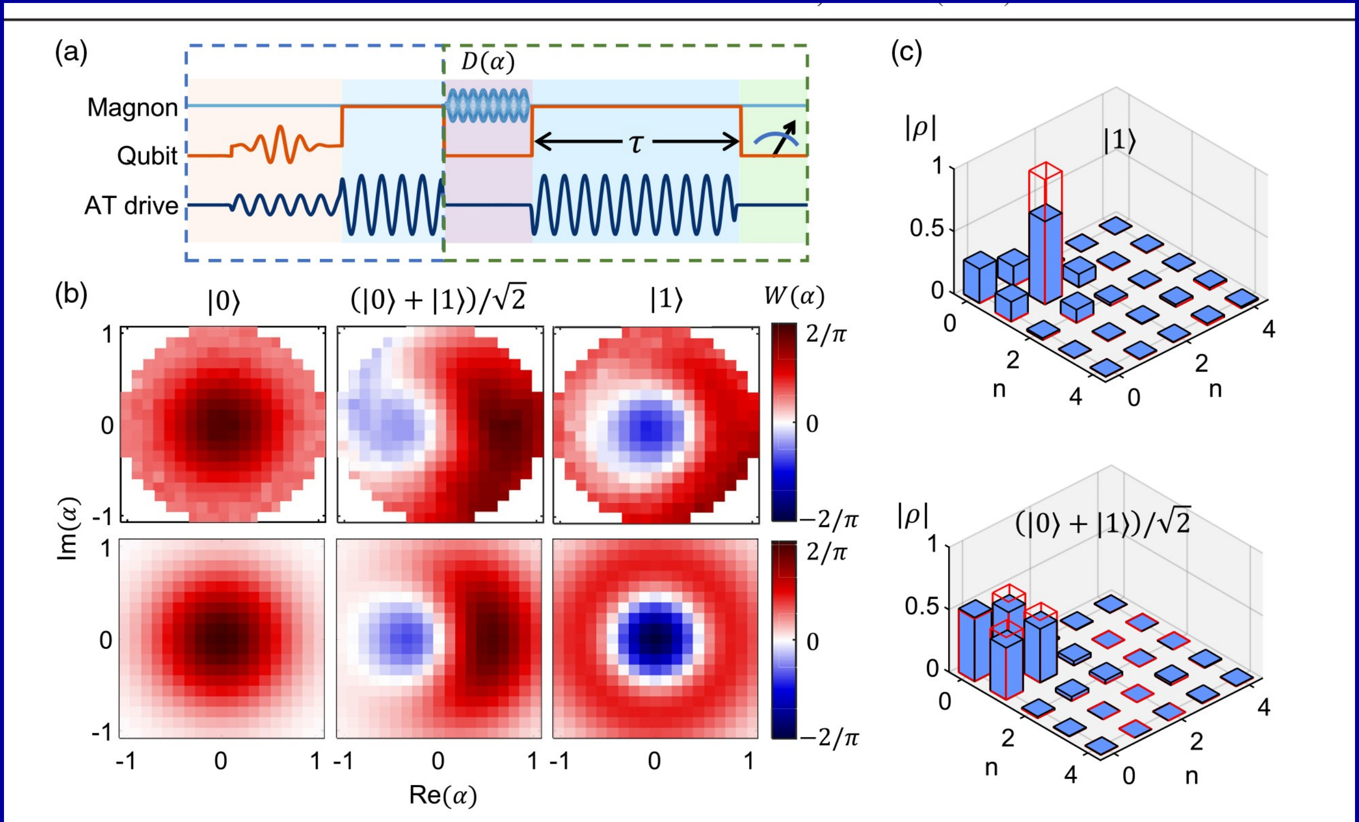



Additions of magnons one by one lead to discrete shifts of the qubit frequency

D. Lachance-Quirion, Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, and Y. Nakamura, Sci. Adv. 3, e1603150 (2017)

# Creation of quantum states

## Resonant coupling regime

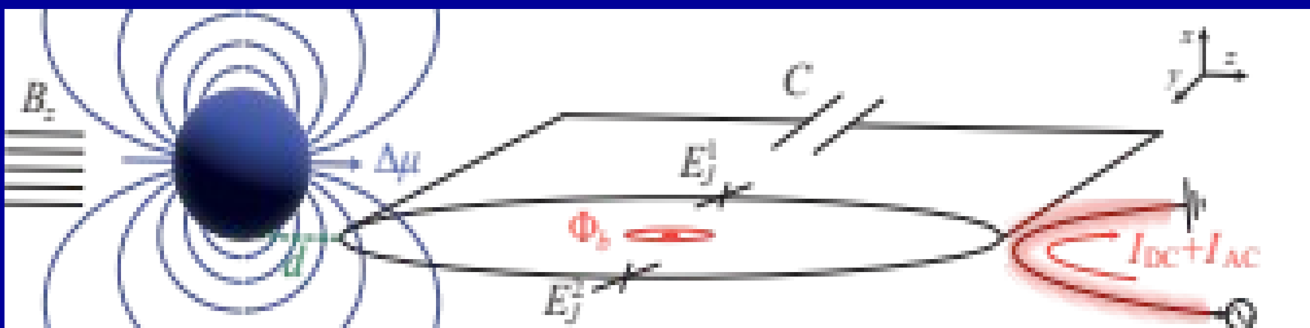



Da Xu, Xu-Ke Gu, He-Kang Li, Yuan-Chao Weng, Yi-Pu Wang, Jie Li, H. Wang, Shi-Yao Zhu, and J. Q. You, Phys. Rev. Lett. **130**, 193603 (2023)

# Magnon-qubit direct coupling

Magnon lifetime: short (100 ns); difficult to create non-trivial quantum states

Idea: Use natural non-linear interactions




$$H_{int} = \hbar J \left( \hat{c}^\dagger \hat{m} + \hat{c}\hat{m}^\dagger \right) + g_{rp} \hat{c}^\dagger \hat{c} \left( \hat{m}^\dagger + \hat{m} \right)$$

"Jaynes-Cummings" coupling

"Magnon radiation pressure" coupling

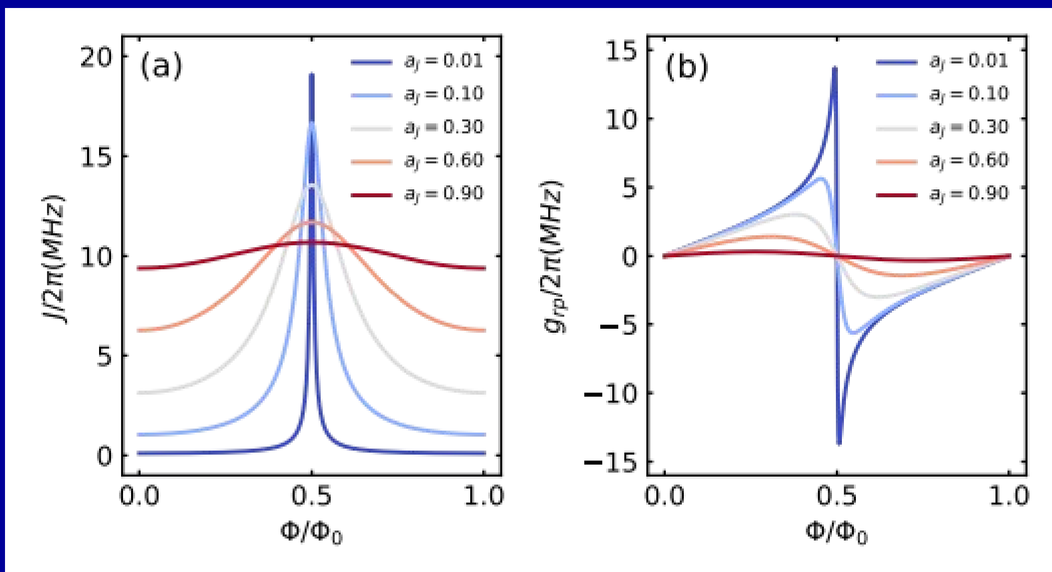
M. Kounalakis, G. E. W. Bauer, and YMB, Phys. Rev. Lett. **109**, 037205 (2022)

# Magnon-qubit coupling

$$H_{int} = \hbar J\left(\hat{c}^\dagger \hat{m} + \hat{c}\hat{m}^\dagger\right) + g_{rp}\hat{c}^\dagger \hat{c}\left(\hat{m}^\dagger + \hat{m}\right)$$

"Jaynes-Cummings" coupling

"Magnon radiation pressure" coupling




$$a_J = \frac{E_{J1} - E_{J2}}{E_J^{\max}}$$

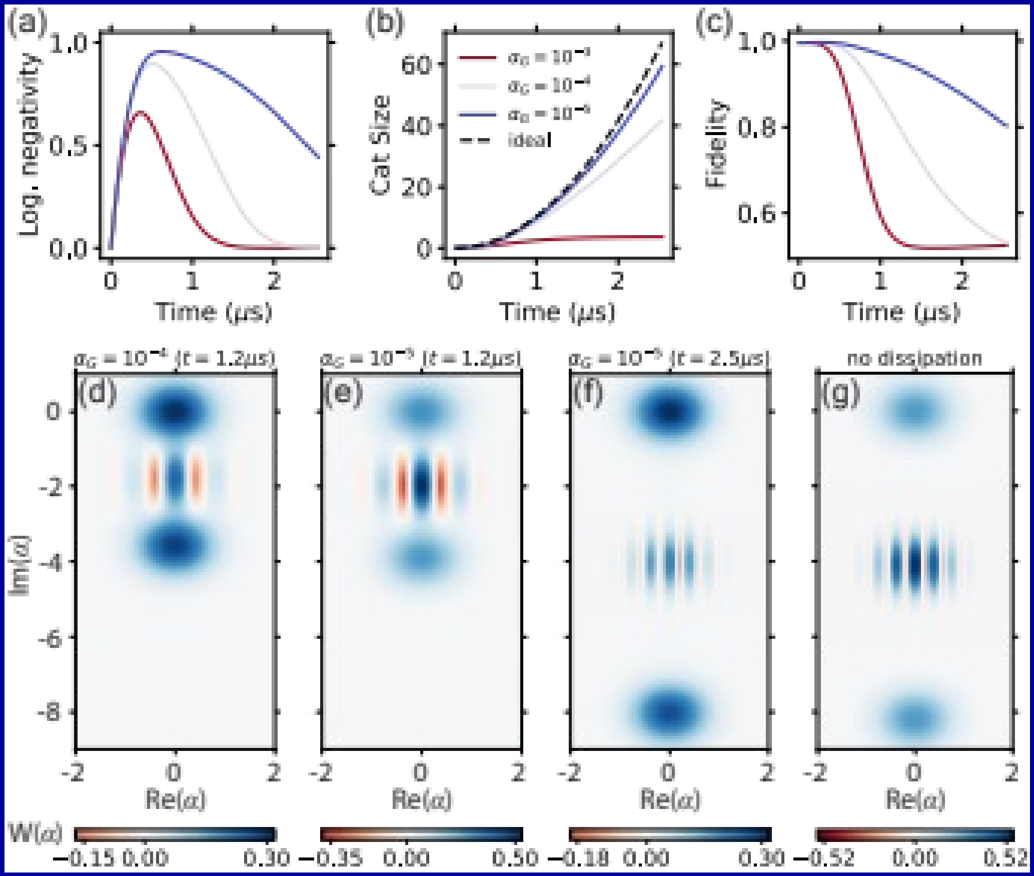
- asymmetry parameter of the transmon

$J \propto a_J$

$$g_{rp} \propto \left(1 - a_J^2\right) \sin\left(2\pi\Phi_b/\Phi_0\right)$$

M. Kounalakis, G. E. W. Bauer, and YMB, Phys. Rev. Lett. **109**, 037205 (2022)

# Generation of cat states




Protocol:

1) Prepare qubit and magnon in the ground states

2) $R_{y,\pi/2}$ on the qubit

3) Time-dependent resonant flux modulation

4) $R_{y,\pi/2}$ on the qubit

5) Projective measurement of the qubit

If |0> is measured the state of the

$$\Psi_{even}\rangle = \frac{1}{N} |0\rangle + |-ig_{rp}\tau\rangle$$

(even cat state)

If |1> is measured the magnon is in the odd cat state

M. Kounalakis, G. E. W. Bauer, and YMB, Phys. Rev. Lett. **109**, 037205 (2022)

Technische Universiteit Delft

# Conclusions

- Spin waves interact with external magnetic field:
  - This interaction can be strong in microwave cavities
  - They facilitate manipulation of spin waves
  - This may result in unusual behavior such as level attraction or chiral propagation
- Quantum properties of magnons can be detected by a qubit
  - One can bring magnons to non-trivial quantum states