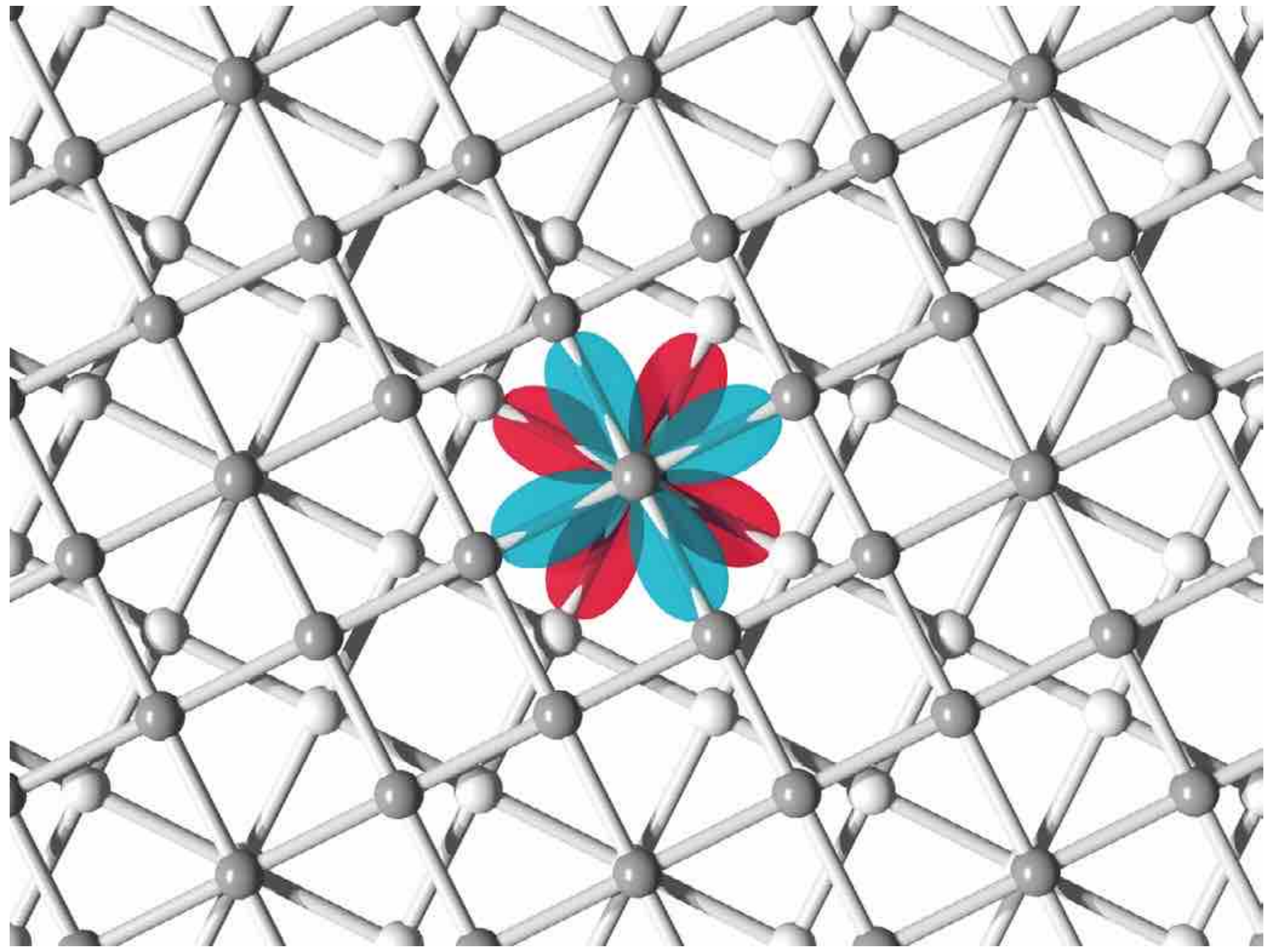
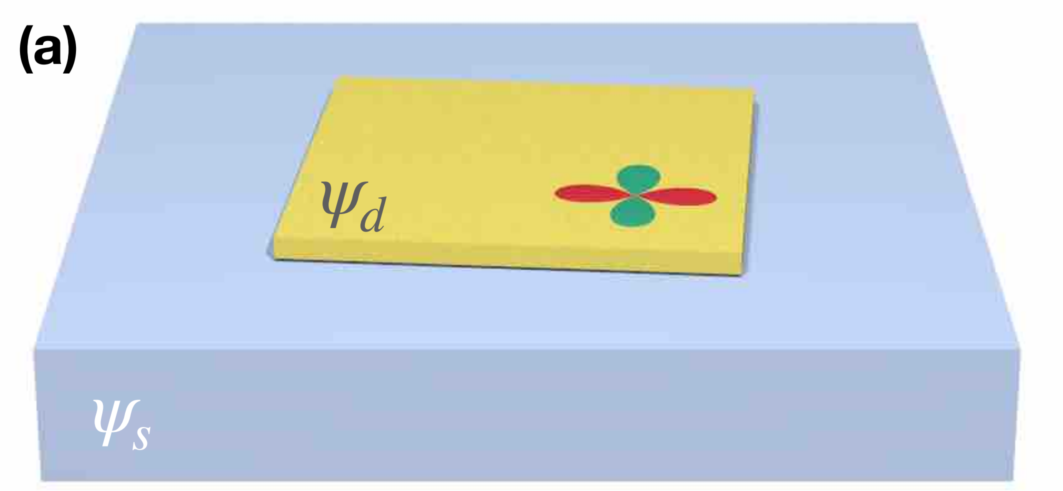
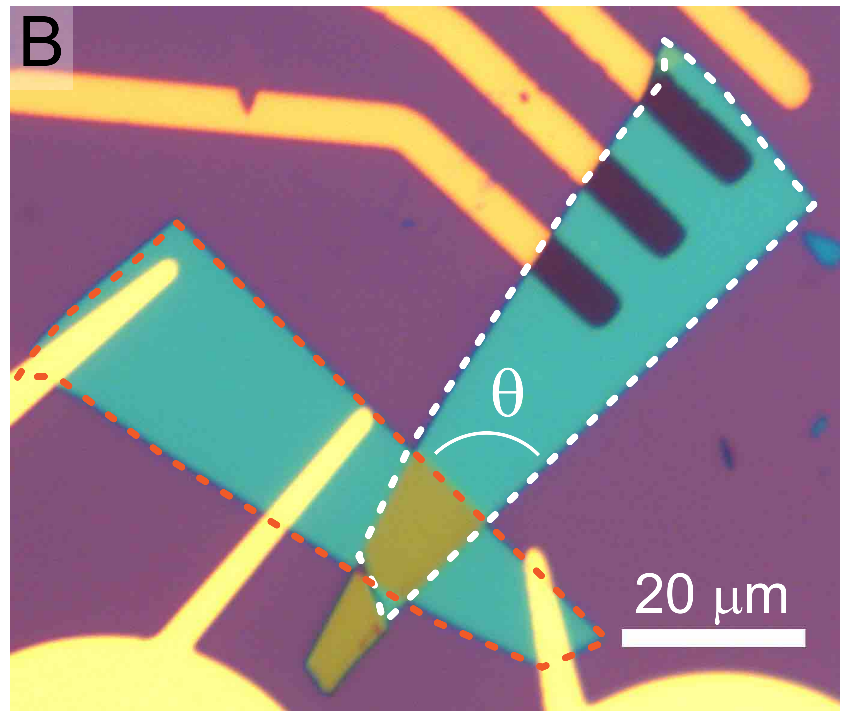
Twisted BSCCO flakes: Applications







M. Franz, September 27, 2023

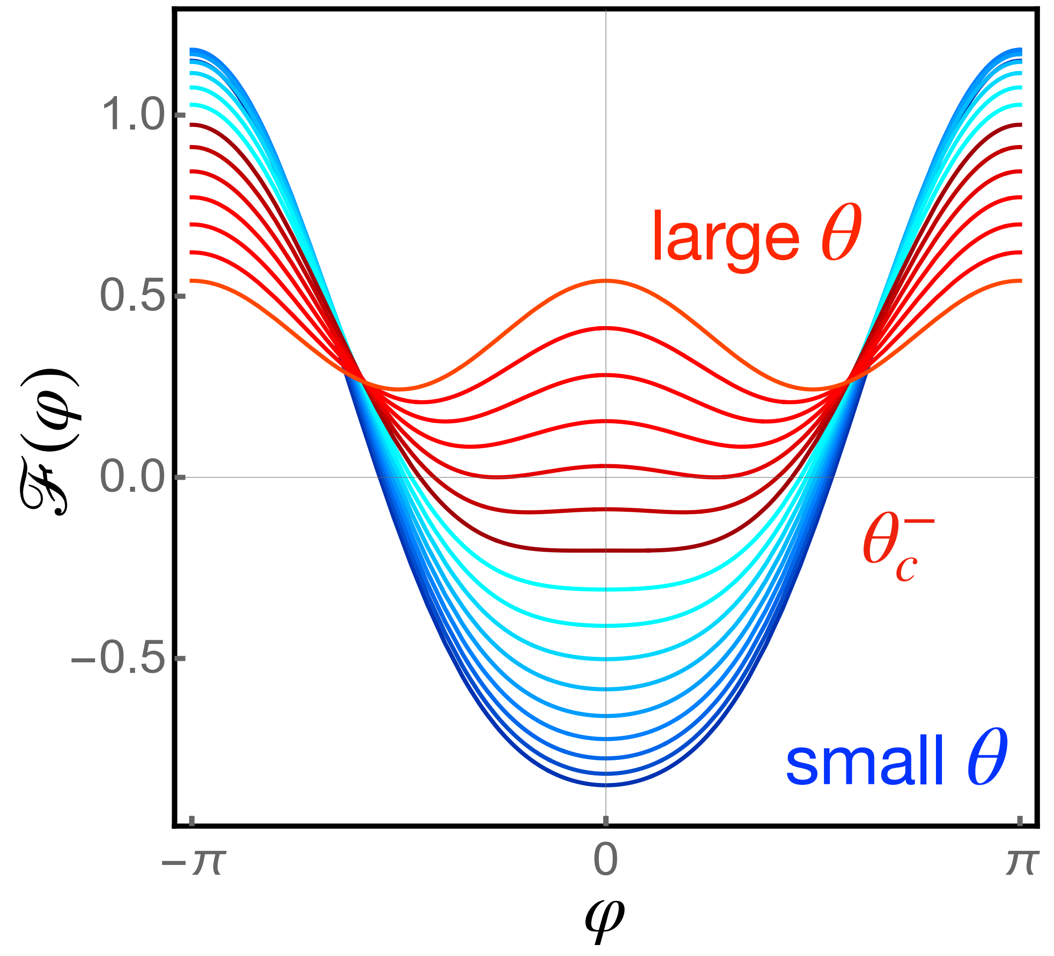
1. Recap: Ginzburg-Landau theory for twisted *d*-wave bilayers

$$\mathcal{F}[\psi_1,\psi_2] = f_0[\psi_1] + f_0[\psi_2] + A\,|\psi_1|^2|\psi_2|^2$$
$$+B(\psi_1\psi_2^* + \mathrm{c.c.}) + C(\psi_1^2\psi_2^{*2} + \mathrm{c.c.})$$

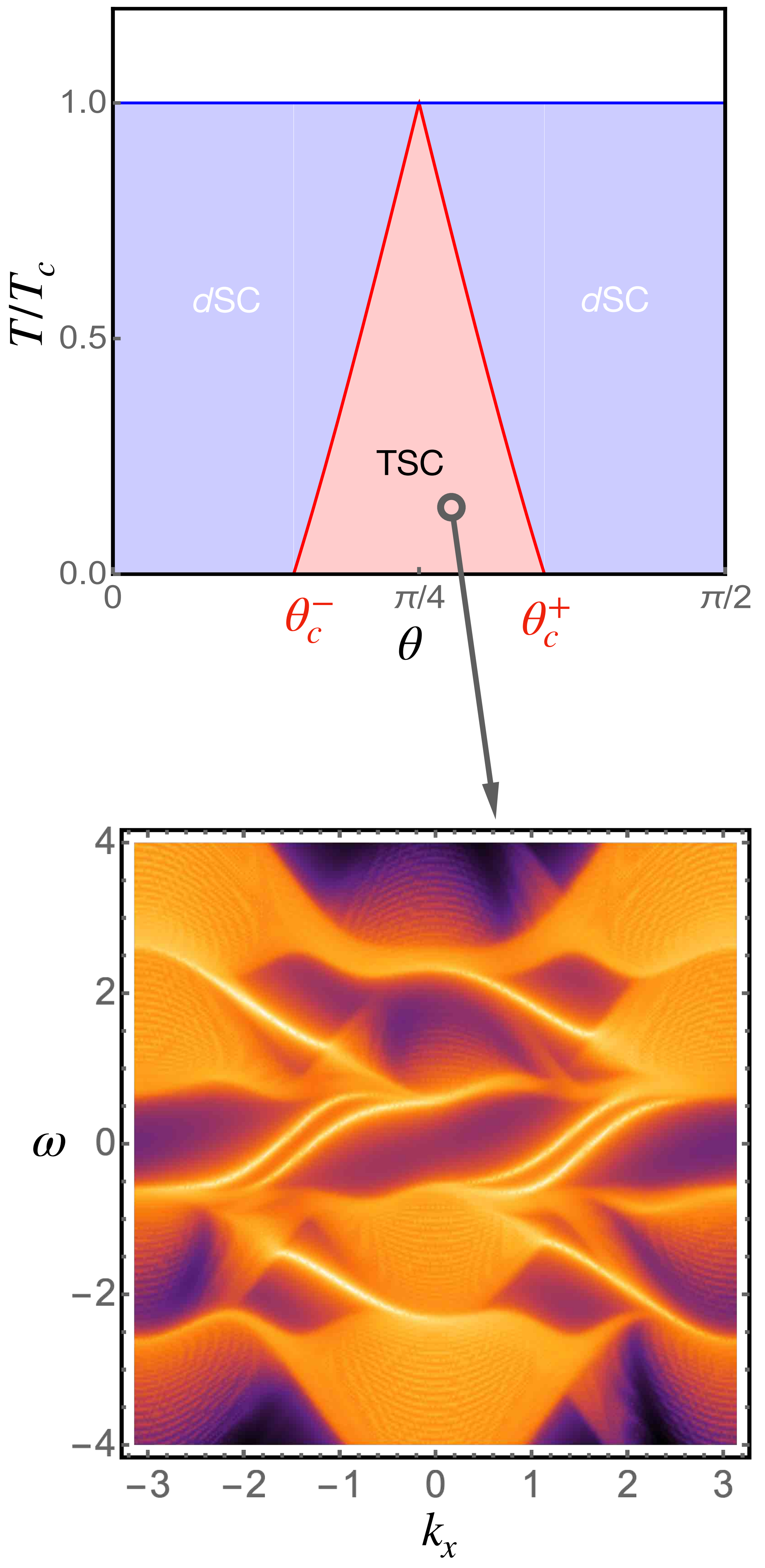
d-wave symmetry dictates $B = -B_0\cos(2\theta)$

Assuming $\psi_1 = \psi, \quad \psi_2 = \psi e^{i\varphi}$ we obtain free energy as a function of the phase

$$\mathcal{F}(\varphi) = \mathcal{F}_0 + 2B_0\psi^2\left[-\cos(2\theta)\cos\varphi + \mathcal{K}\cos(2\varphi)\right]$$



$$\mathcal{K} = C\psi^2/B_0$$



Applications: Majorana Fermions?

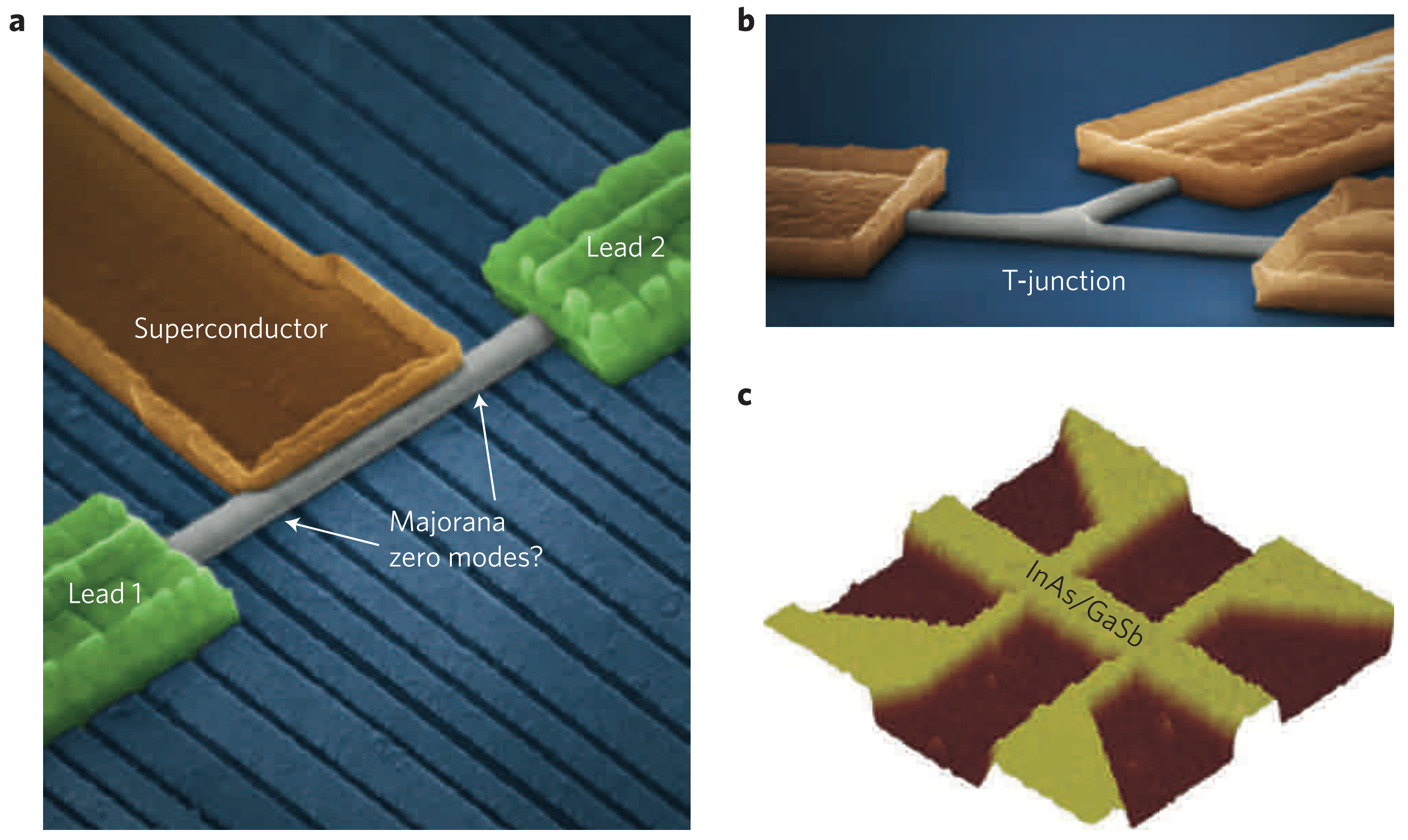
- Topological superconductors often host Majorana zero modes at vortices, corners, or other defects. **Are there such zero modes in a $d_{x^2-y^2} + id_{xy}$ superconductor?**

- Because of spin degeneracy Majorana zero modes are not expected to appear in a $d_{x^2-y^2} + id_{xy}$ superconductor.
- Can we think of a variant on the construction that could host Majoranas?

Majorana modes materialize

Condensed-matter physicists are steadily closing in on exotic excitations known as Majorana modes that could advance both fundamental science and quantum computing.

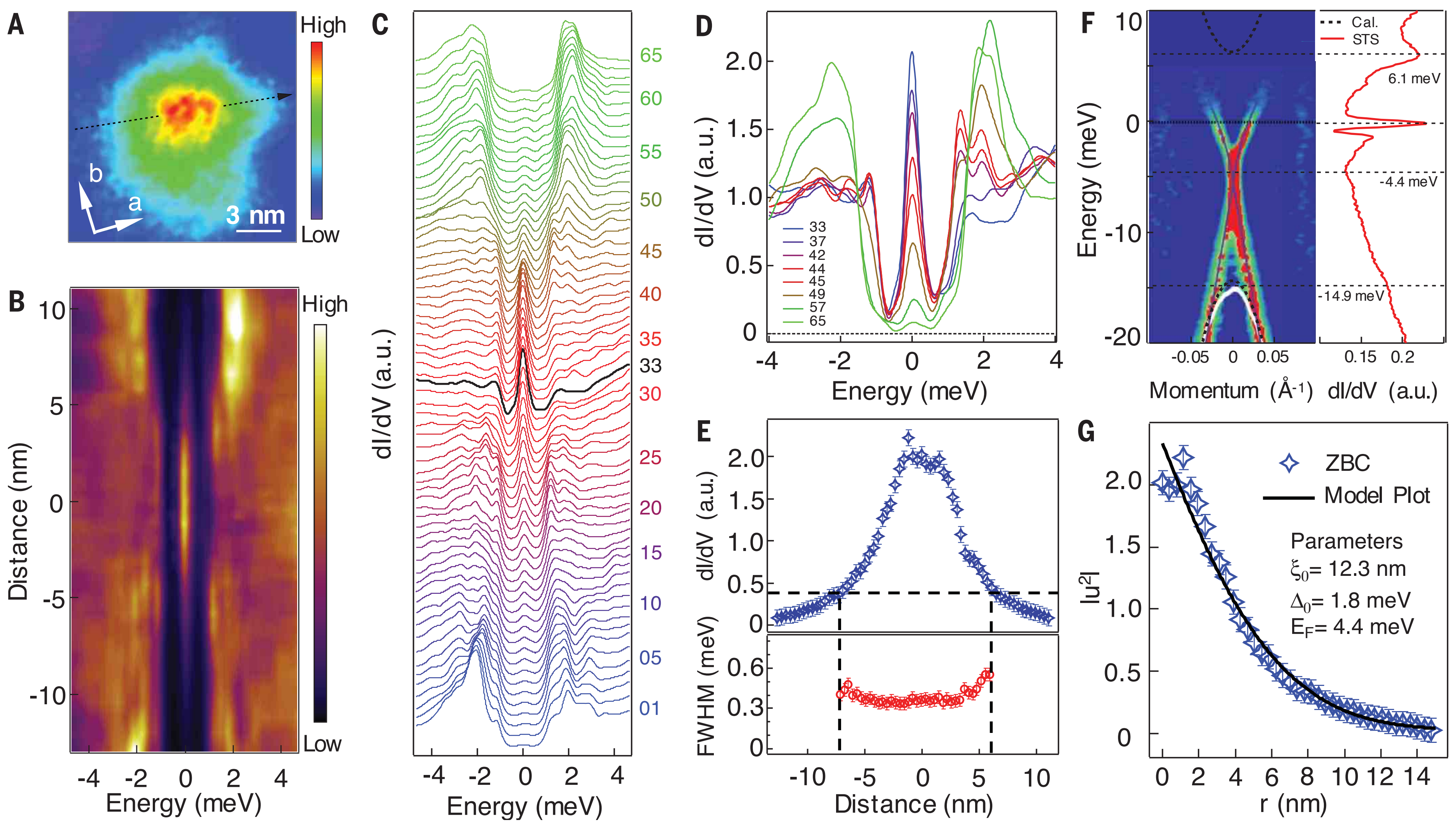
Jason Alicea



TOPOLOGICAL MATTER

Evidence for Majorana bound states in an iron-based superconductor

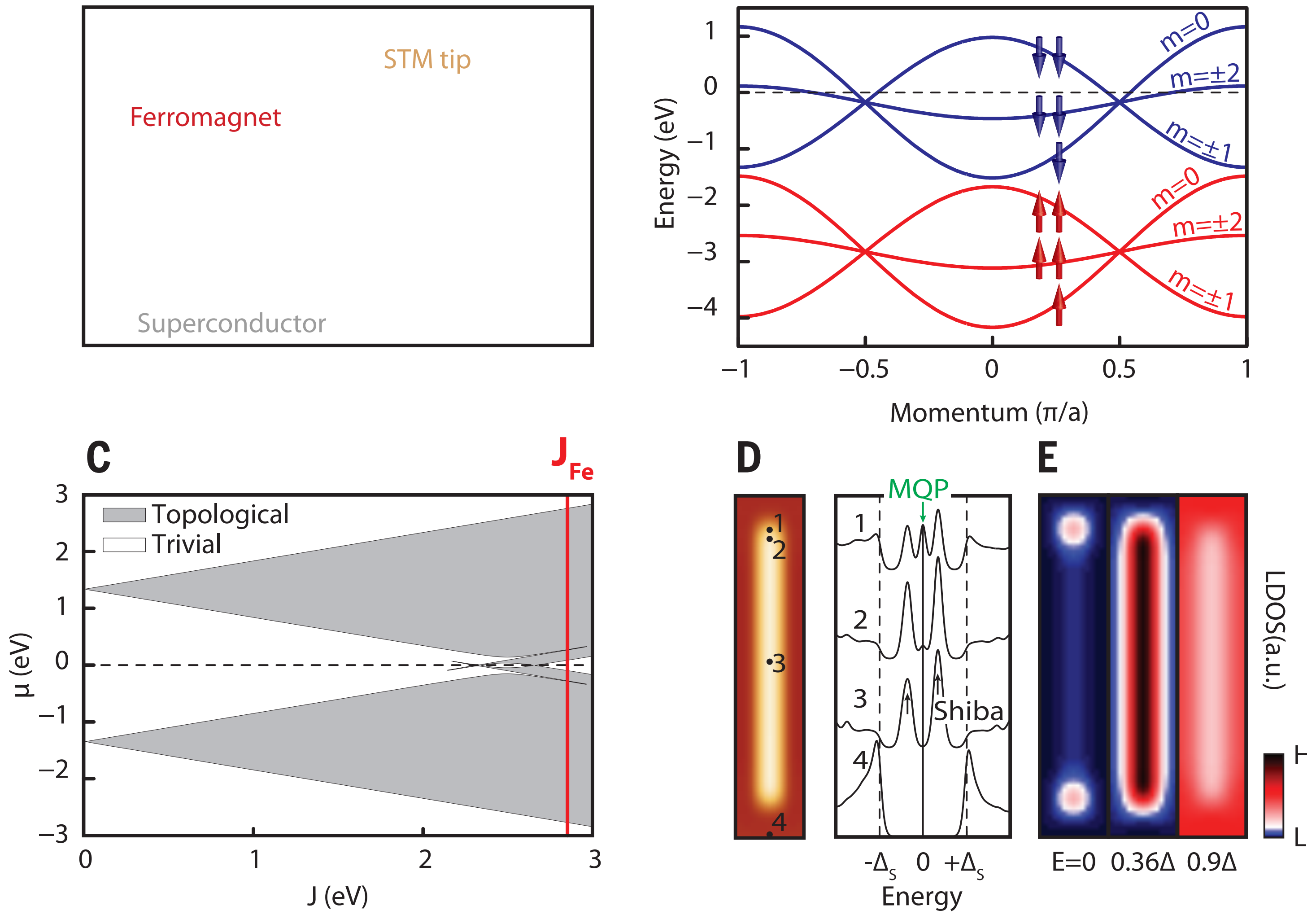
Dongfei Wang[1,2]*, Lingyuan Kong[1,2]*, Peng Fan[1,2]*, Hui Chen[1], Shiyu Zhu[1,2], Wenyao Liu[1,2], Lu Cao[1,2], Yujie Sun[1,3], Shixuan Du[1,3,4], John Schneeloch[5], Ruidan Zhong[5], Genda Gu[5], Liang Fu[6], Hong Ding[1,2,3,4]†, Hong-Jun Gao[1,2,3,4]†



TOPOLOGICAL MATTER

Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor

Stevan Nadj-Perge,[1]* Ilya K. Drozdov,[1]* Jian Li,[1]* Hua Chen,[2]* Sangjun Jeon,[1] Jungpil Seo,[1] Allan H. MacDonald,[2] B. Andrei Bernevig,[1] Ali Yazdani[1]†



Majorana idea 1: Twisted cuprate bilayer on top of a topological insulator

Variation on the Fu-Kane construction

- Twisted cuprate bilayer supplies fully gapped high-T_c superconductivity
- TI surface provides the requisite spin-orbit coupling
- Will this setup host Majorana zero mode in the vortex?

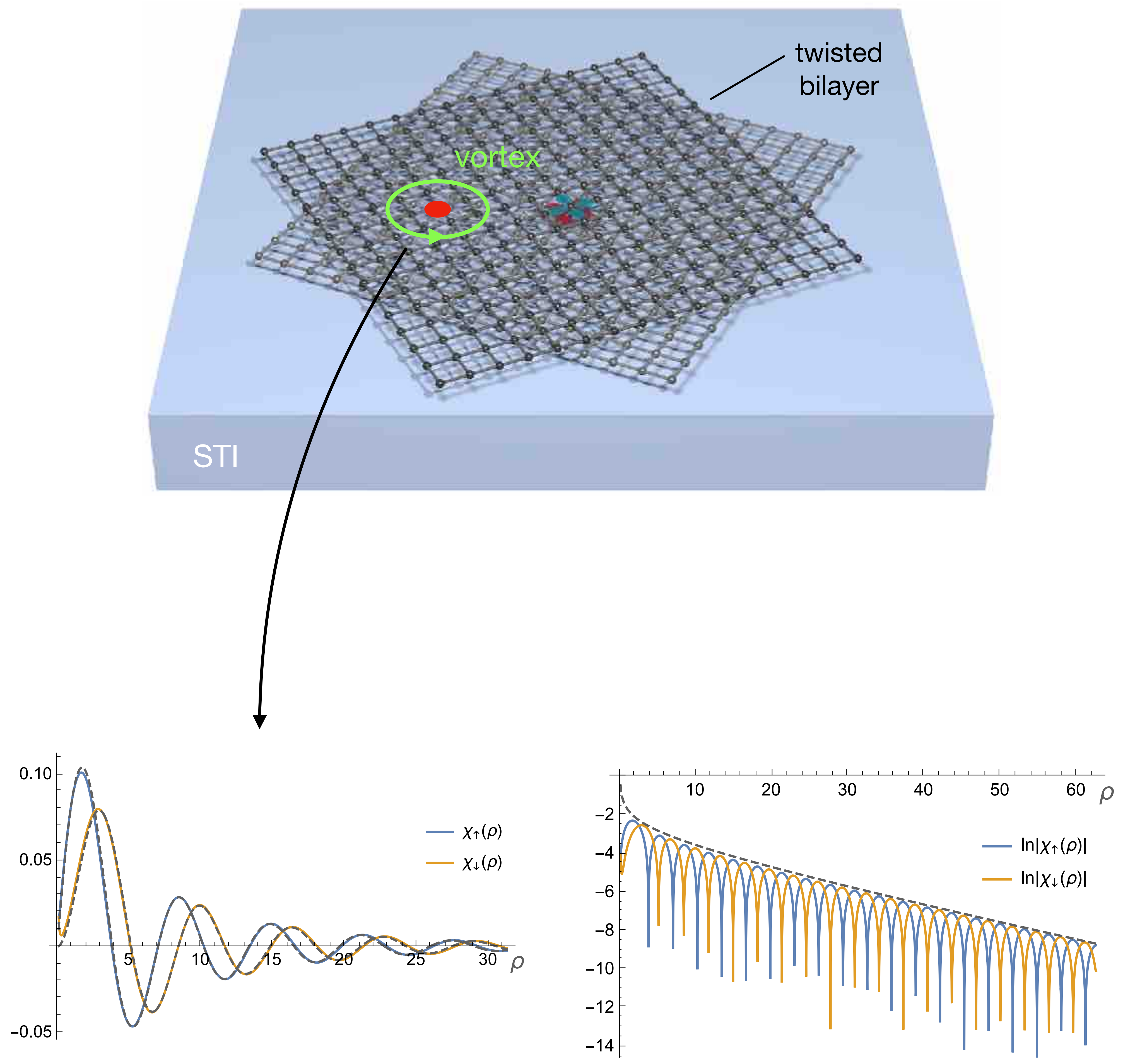
- We solve the problem by an approximate analytic approach and exact numerical diagonalization
- Clear evidence for MZM bound to the vortex core

$$H = \begin{pmatrix} v\boldsymbol{\sigma}\cdot\boldsymbol{p} - \mu & \hat{\Delta} \\ \hat{\Delta}^{\dagger} & -v\boldsymbol{\sigma}\cdot\boldsymbol{p} + \mu \end{pmatrix}$$

$$\mu\chi_{\uparrow} - \left[v\left(\partial_r + \frac{l+1}{r}\right) + \hat{\Delta}_{l,l+1}\right]\chi_{\downarrow} = 0,$$

$$\mu\chi_{\downarrow} + \left[v\left(\partial_r - \frac{l}{r}\right) + \hat{\Delta}_{l+1,l}\right]\chi_{\uparrow} = 0,$$

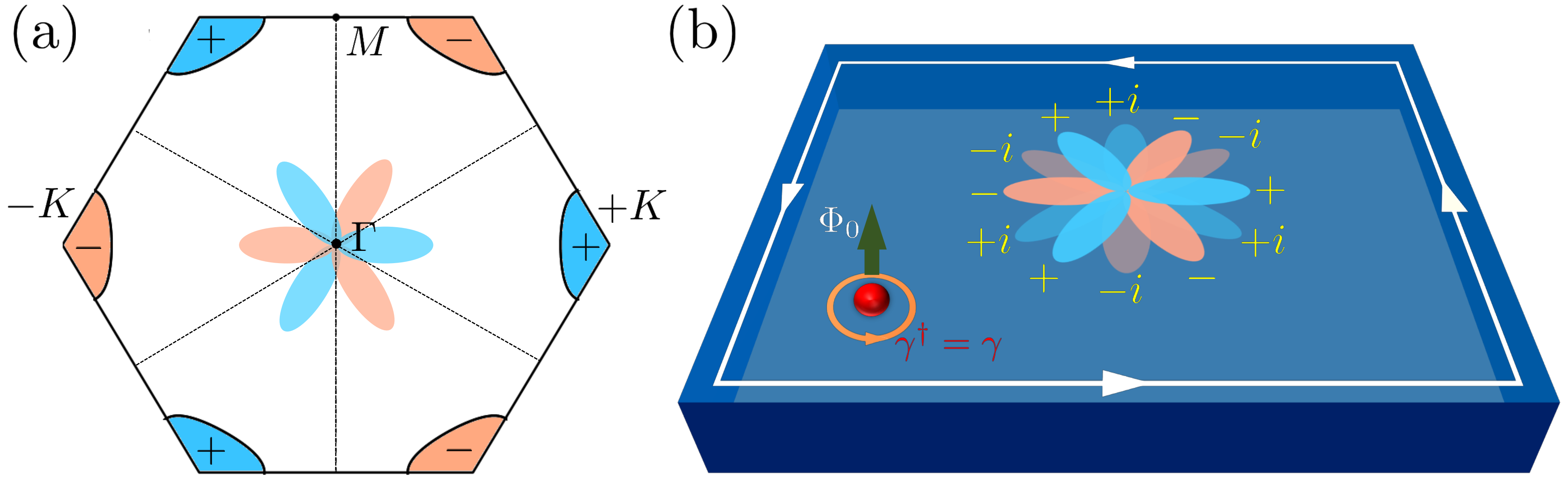
[Mercado, Sahoo and Franz, PRL 128, 137002 (2022).]



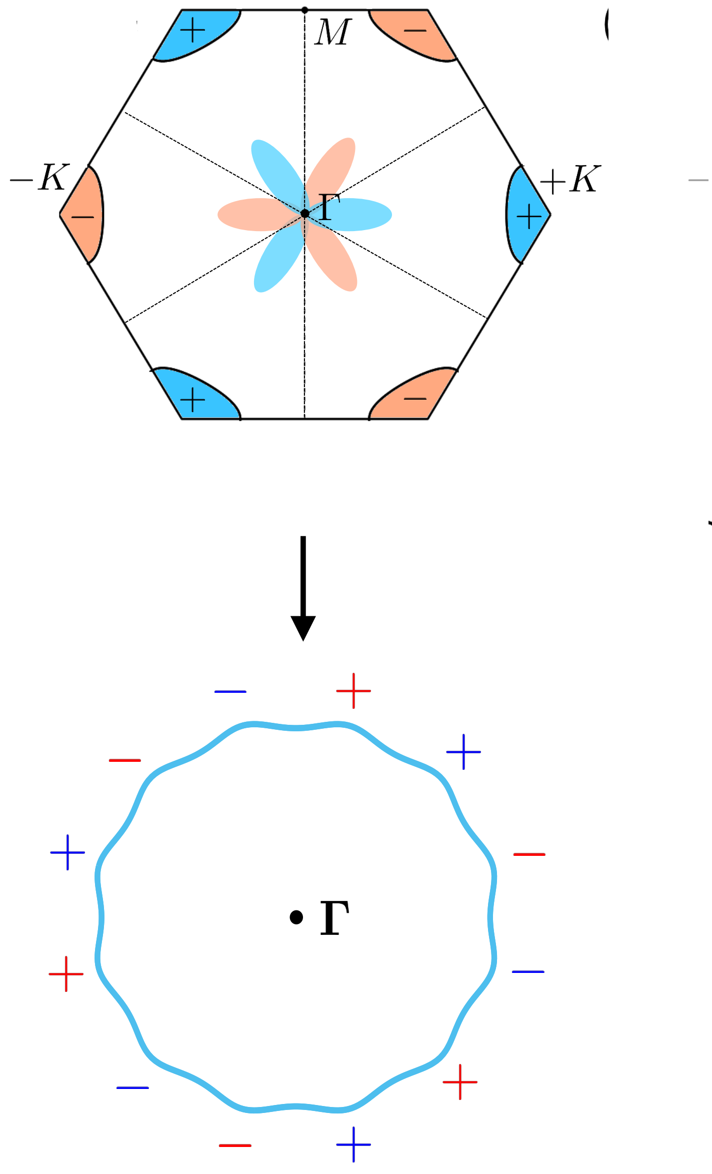
Majorana idea 2: Maximally twisted double-layer spin-triplet valley-singlet superconductors

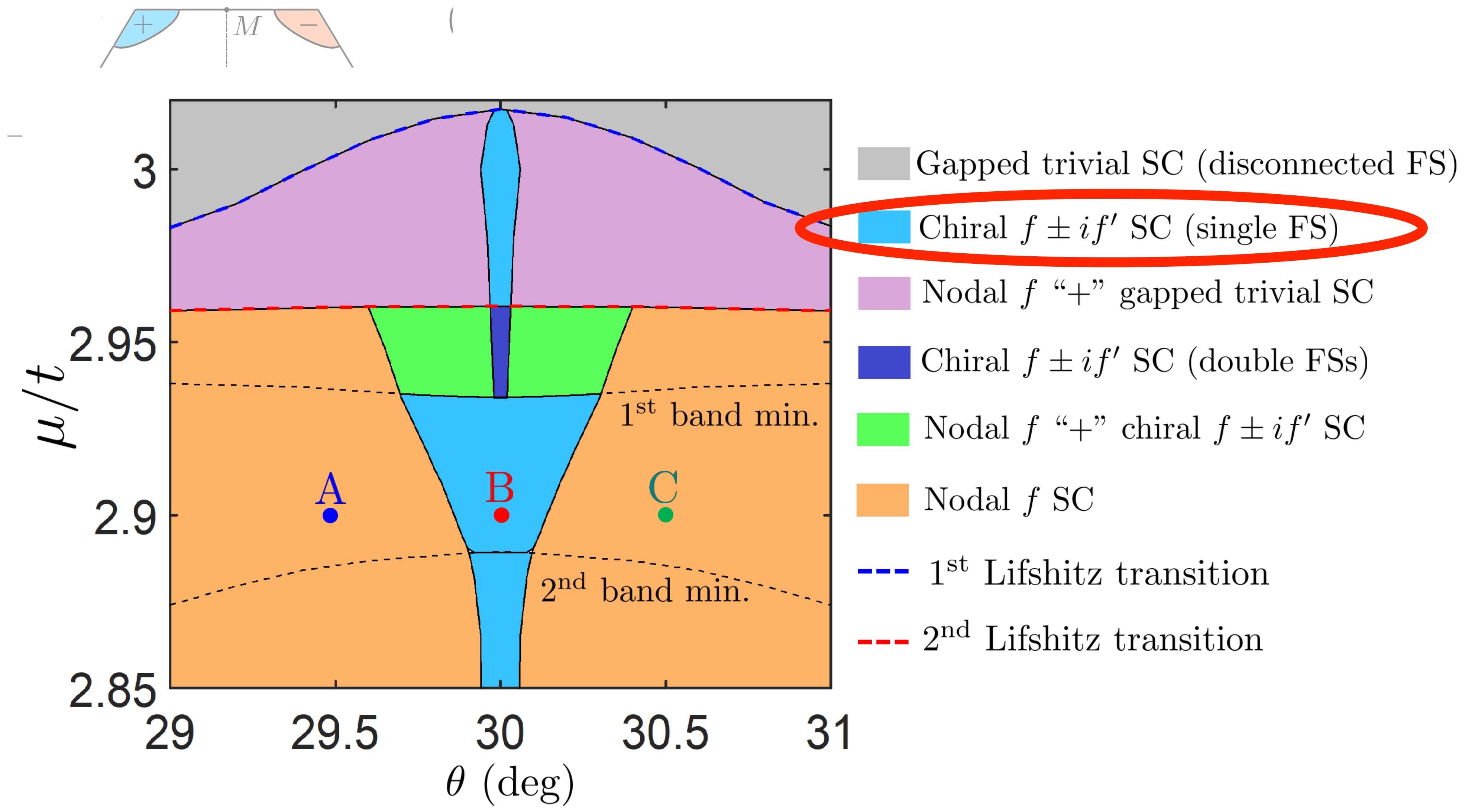
[Comm. Physics 6 (1), 47 (2023); arXiv:2206.05599]

- Rhombohedral trilayer graphene, Bernal bilayer graphene and ZrNCl are thought to be spin-triplet valley-singlet superconductors with f-wave order parameter.
- We consider a bilayer formed of such a STVS material close to 'maximal' twist angle of 30^0.



[Zhou, Egan, Kush, Franz, arXiv:2206.05599]

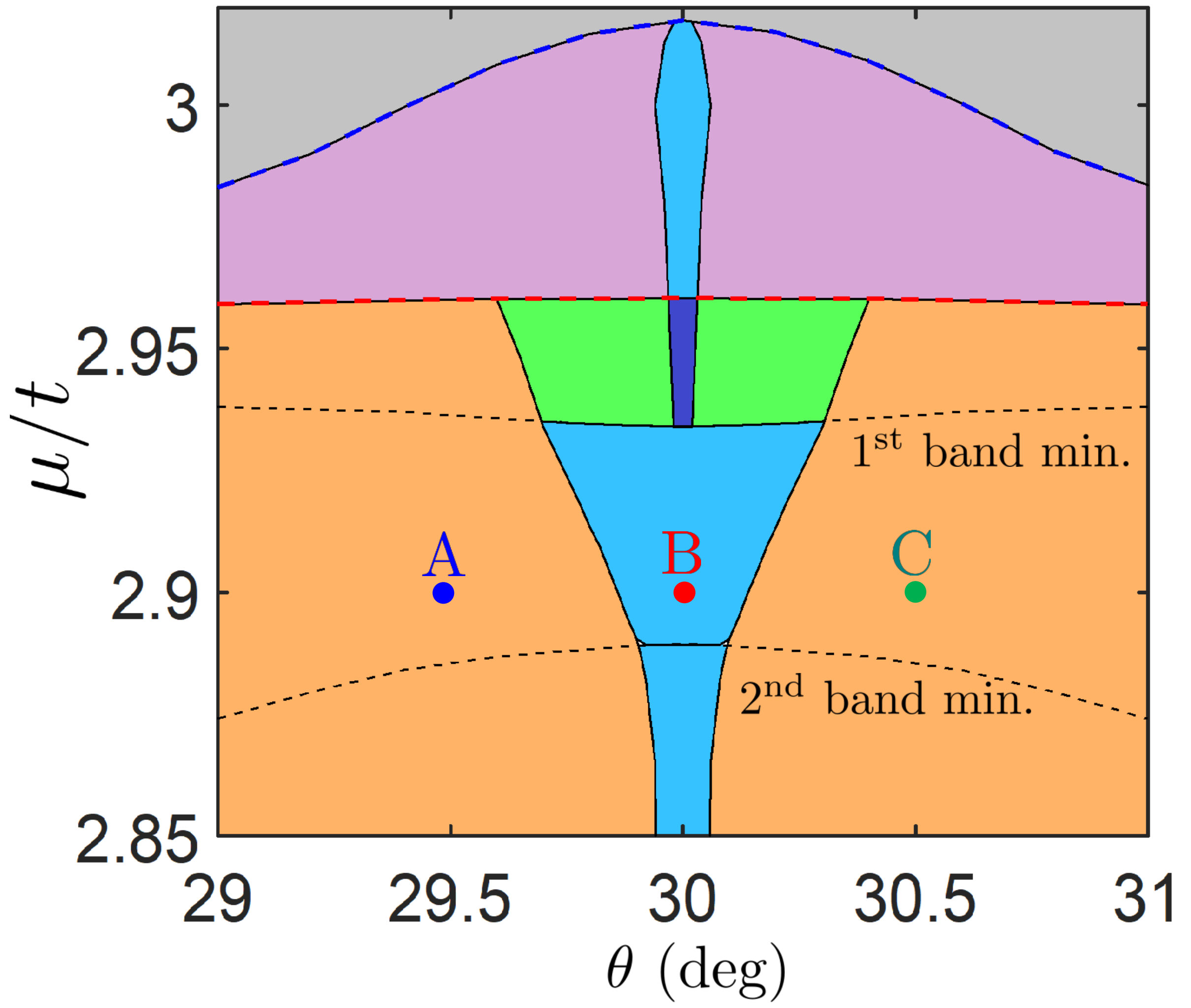


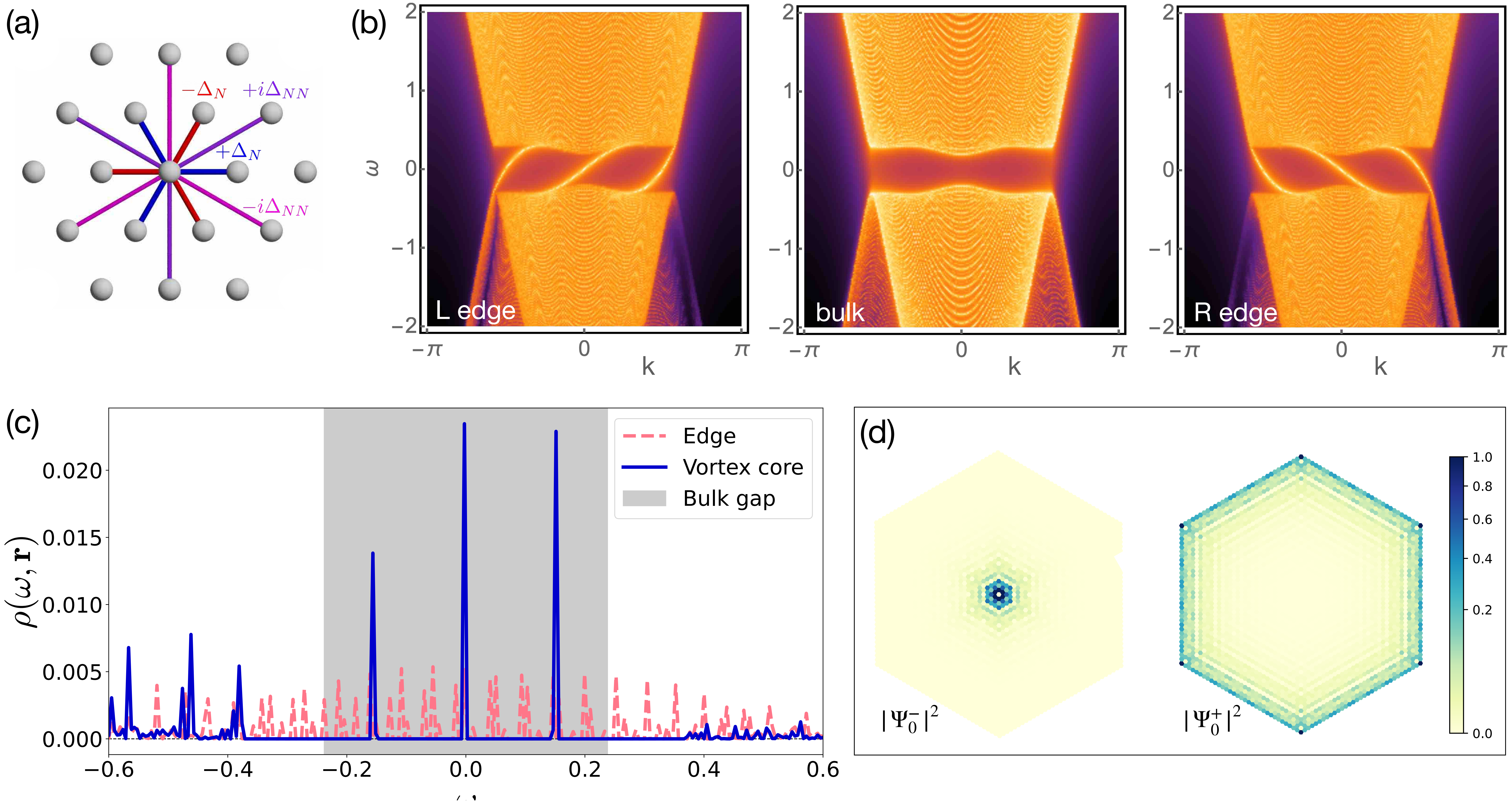


- For a range of electron density and twist angles the combined system forms a spontaneously T-broken phase with $f + if'$ order parameter.
- This chiral phase exhibits non-Abelian topology: it hosts an odd number of chiral Majorana edge modes and a single Majorana zero mode in the vortex core.

Majorana idea 2: Maximally twisted double-layer spin-triplet valley-singlet superconductors

[Comm. Physics 6 (1), 47 (2023); arXiv:2206.05599]





Numerical simulations in a minimal model of $f + if'$ superconductor confirm the presence of:

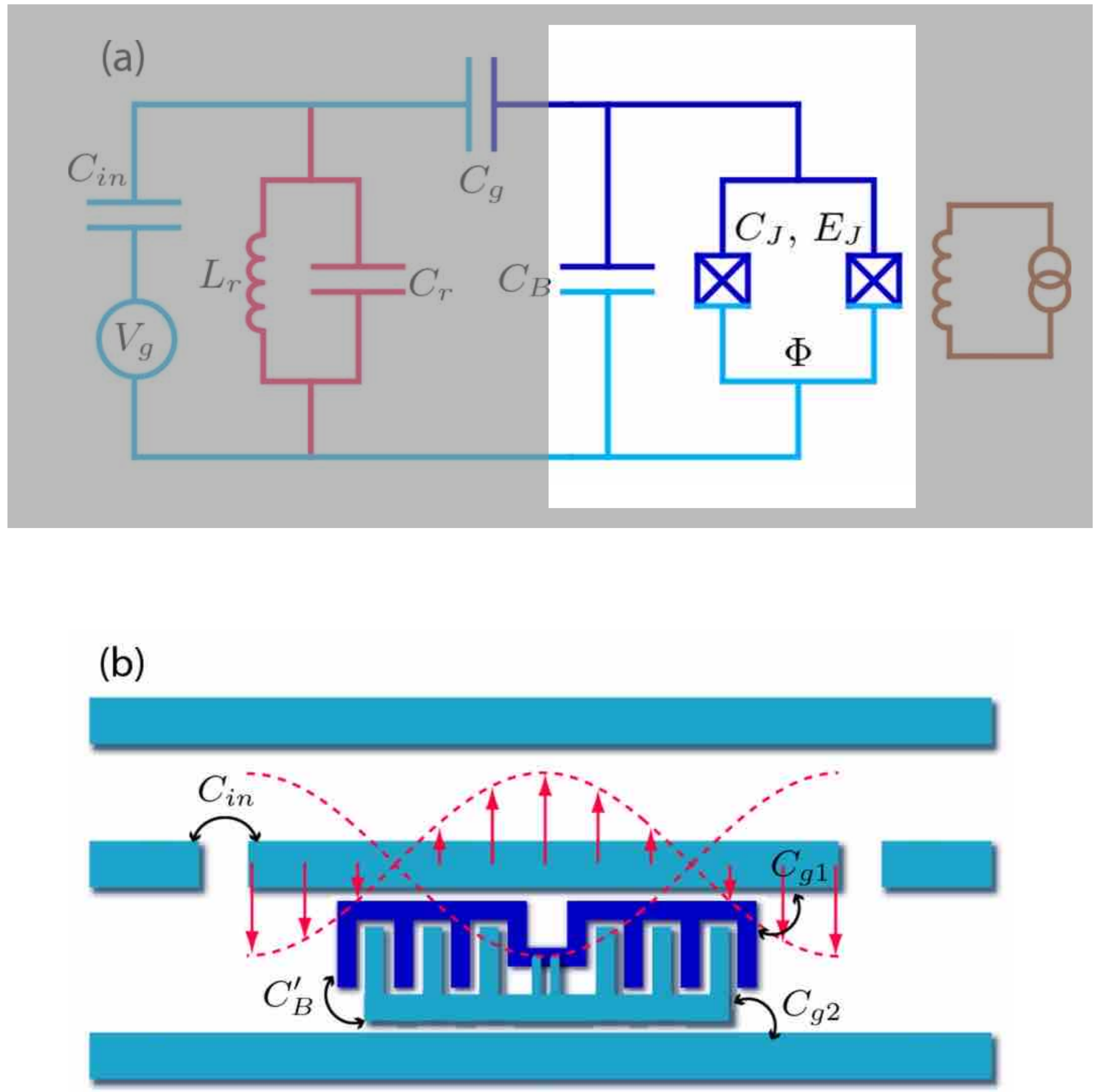
1. Odd number of chiral Majorana edge modes at the sample boundary
2. Unpaired Majorana zero modes bound to each vortex

Another application: Improved transmon qubit

[arXiv:2308.02547]

Brief Review:

The original transmon qubit



Koch et al., PHYSICAL REVIEW A **76**, 042319 (2007)

- Transmon architecture was developed to overcome the "offset charge" problem of earlier qubit variants (e.g. Cooper pair box)

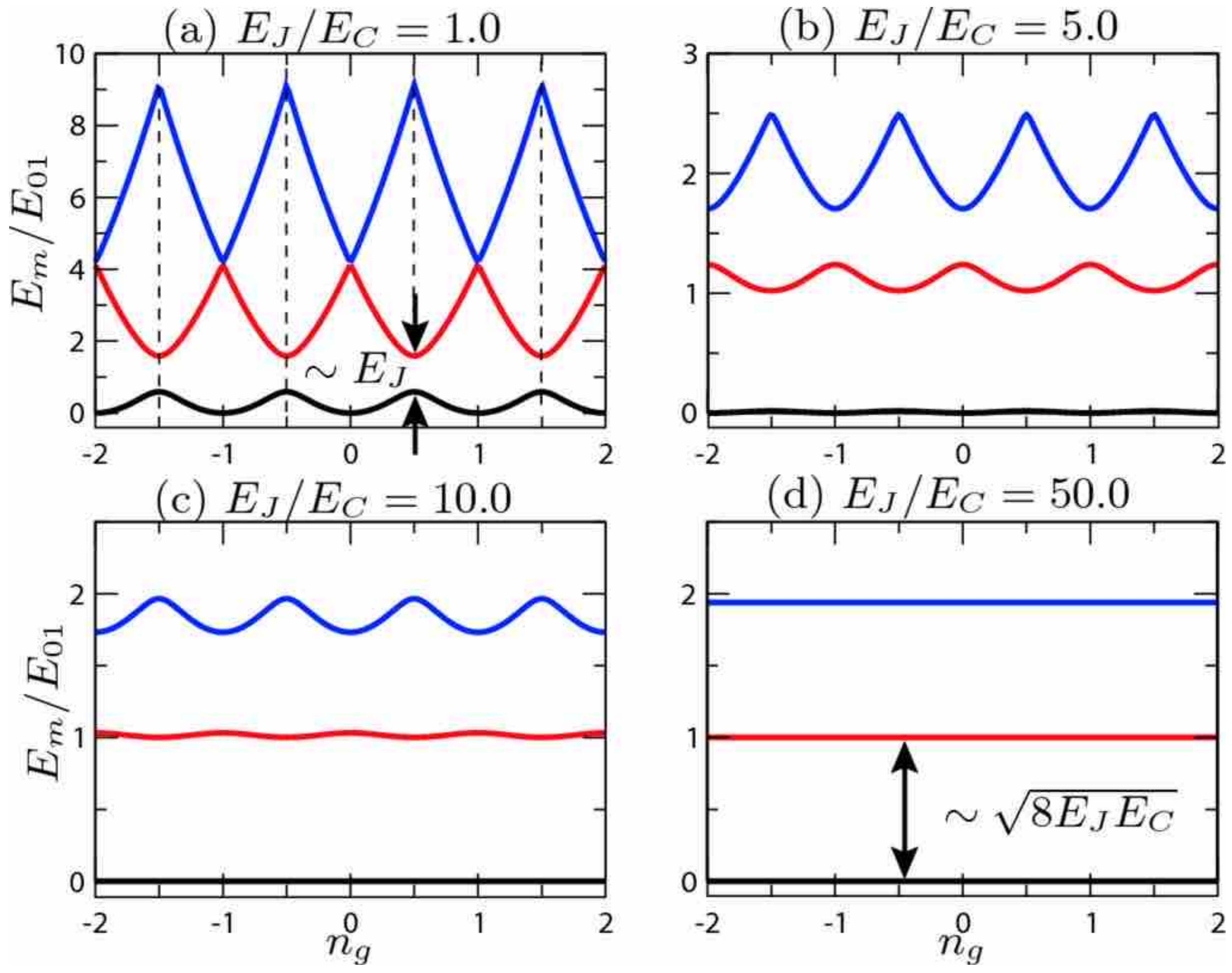
Transmon Schrodinger equation:

$$\left[4E_C\left(-i\frac{d}{d\varphi}-n_g\right)^2-E_J\cos\varphi\right]\psi(\varphi)=E\psi(\varphi),$$

- Here $E_C = e^2/2C$ is the charging energy
- $\hat{n} = -i\frac{d}{d\varphi}$ is the Cooper pair number operator
- n_g denotes the uncontrolled offset charge

Transmon Energy spectrum

$$\left[4E_C\left(-i\frac{d}{d\varphi}-n_g\right)^2-E_J\cos\varphi\right]\psi(\varphi)=E\psi(\varphi),$$



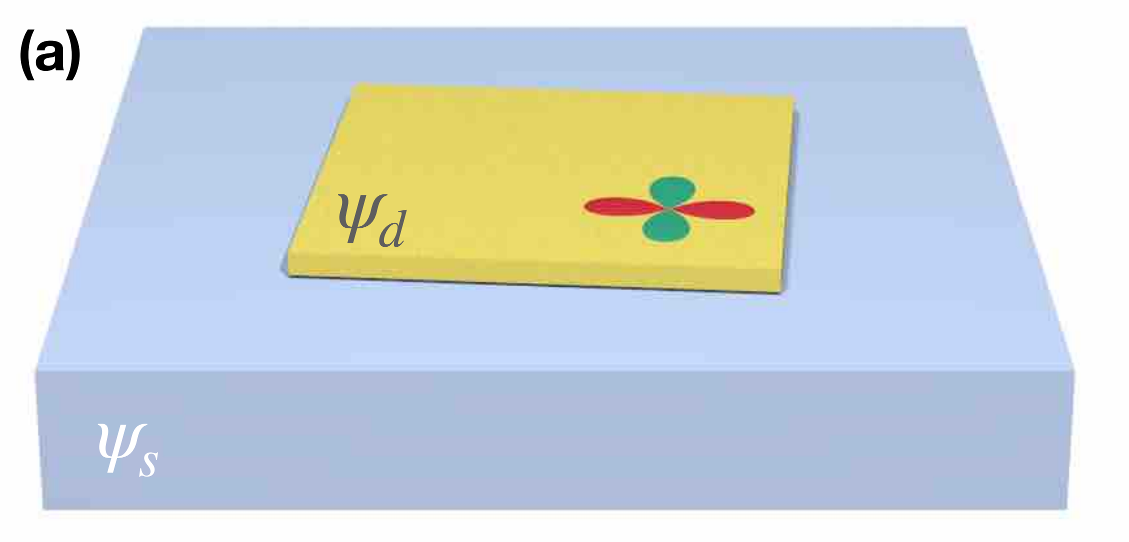
Conclusion:

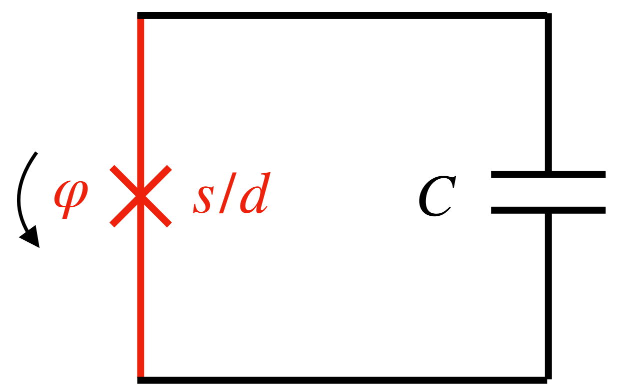
- Transmon qubit in the limit $E_J \gg E_C$ becomes insensitive to the offset charge fluctuations
- At the same time it exhibits robustness against other types of noise (e.g. flux).
- A distinct disadvantage is the weak anharmonicity of its energy spectrum that places limits on the maximum speed of operation.

d-mon: transmon with strong anharmonicity

[arXiv:2308.02547]

$$\begin{aligned} F[\psi_s,\psi_d] &= F_s[\psi_s] + F_d[\psi_d] + A|\psi_s|^2|\psi_d|^2 \\ &\quad + B(\psi_s\psi_d^* + \text{c.c.}) + C(\psi_s^2\psi_d^{*2} + \text{c.c.}) \end{aligned}$$





- When both superconductors respect the C4 rotation symmetry then *B* must vanish.
- This is because under C4 we have $\psi_s \to \psi_s$ but $\psi_d \to -\psi_d$.
- If the C4 symmetry is weakly broken (as happens in BSCCO) then we expect small nonzero *B*.

The Josephson energy thus becomes (for *B*=0):

$$F(\varphi) = F_0 + 2C|\psi_s|^2|\psi_d|^2\cos 2\varphi$$

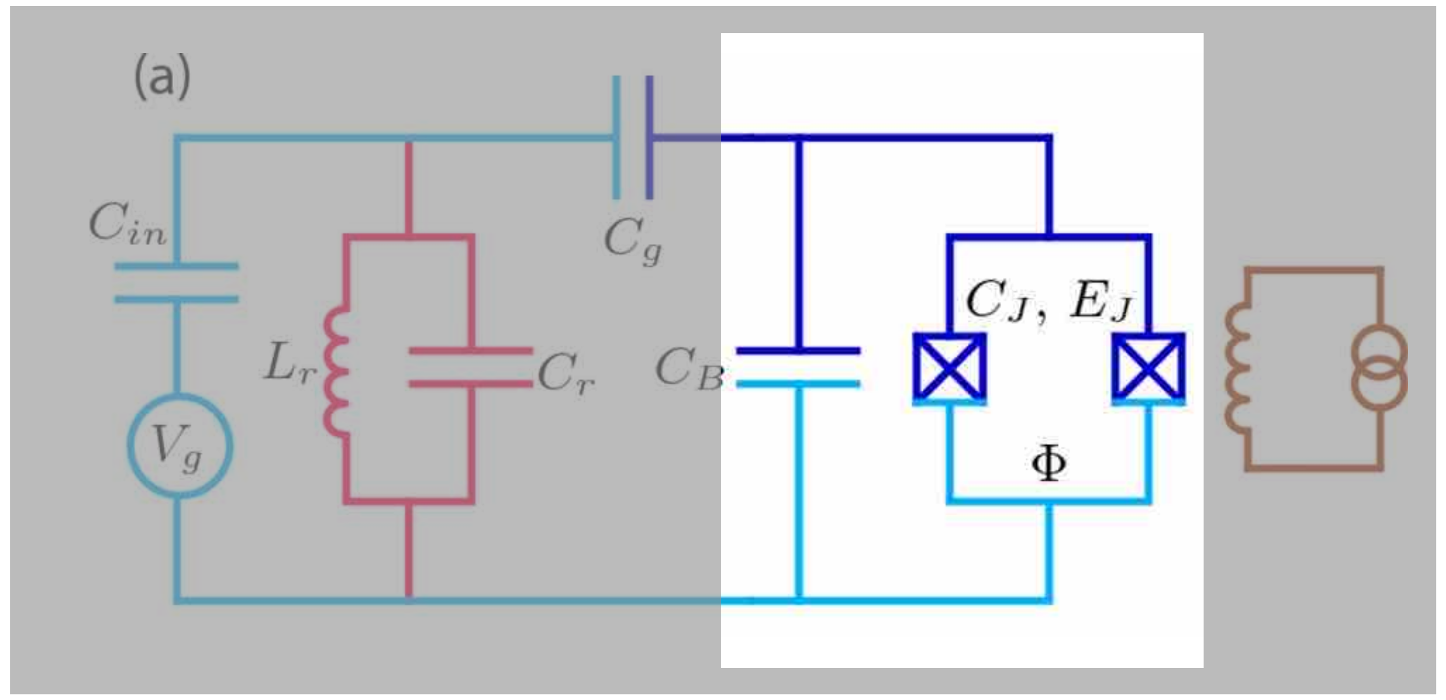
d-mon Schrodinger equation:

$$\left[4E_C\left(-i\frac{d}{d\varphi} - n_g\right)^2 - E_J\cos \underset{\uparrow}{2\varphi}\right]\psi(\varphi) = E\psi(\varphi)$$

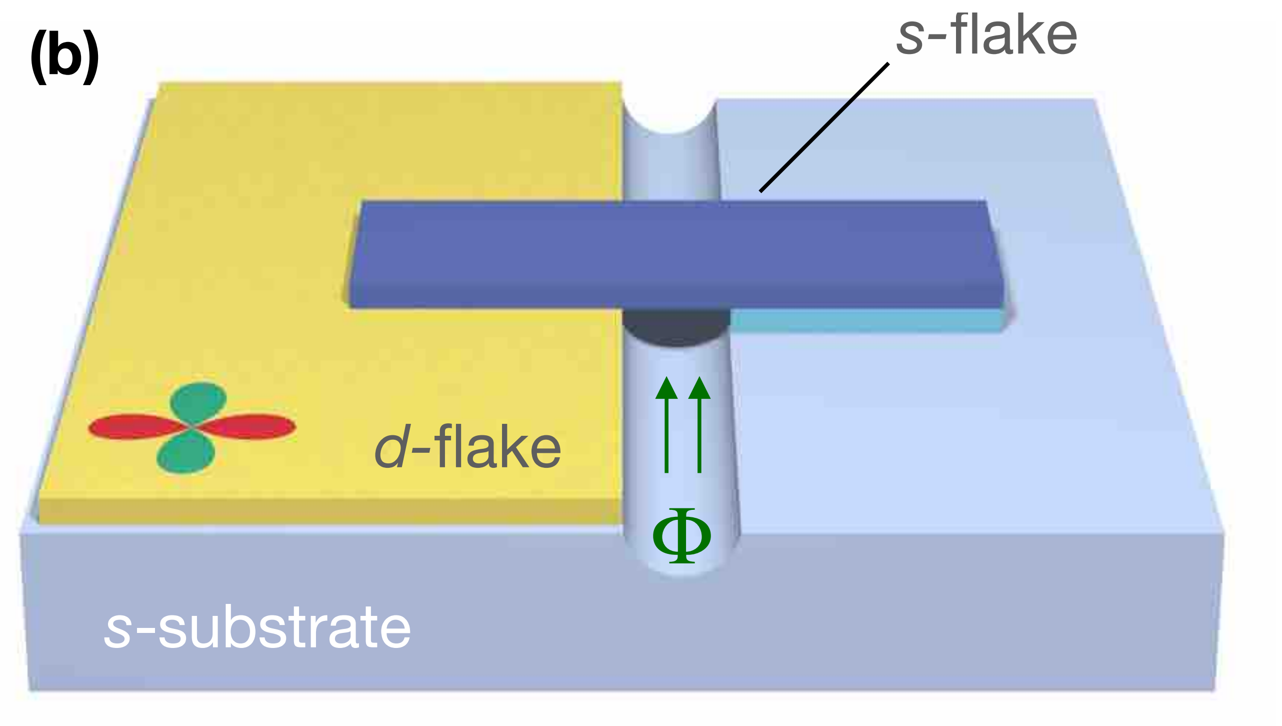
Split *d*-mon: a practical tunable qubit

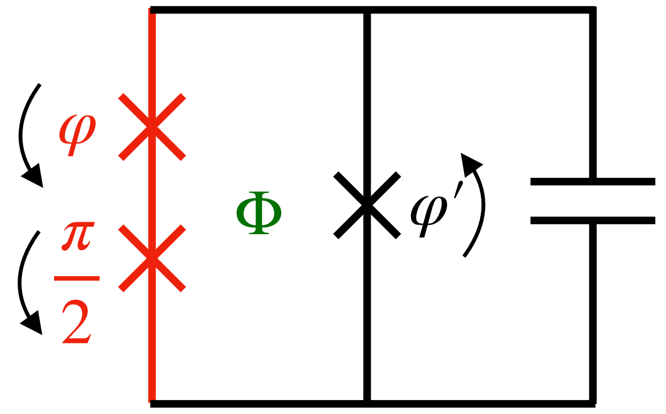
[arXiv:2308.02547]

Koch et al., PHYSICAL REVIEW A **76**, 042319 (2007)



“split” transmon





Split *d*-mon Schrodinger equation:

$$\left[4E_C\left(-i\frac{d}{d\varphi}-n_g\right)^2-E_J\cos 2\varphi+E_S\cos(\varphi-\phi_{\text{ex}})\right]\psi(\varphi)=E\psi(\varphi),$$

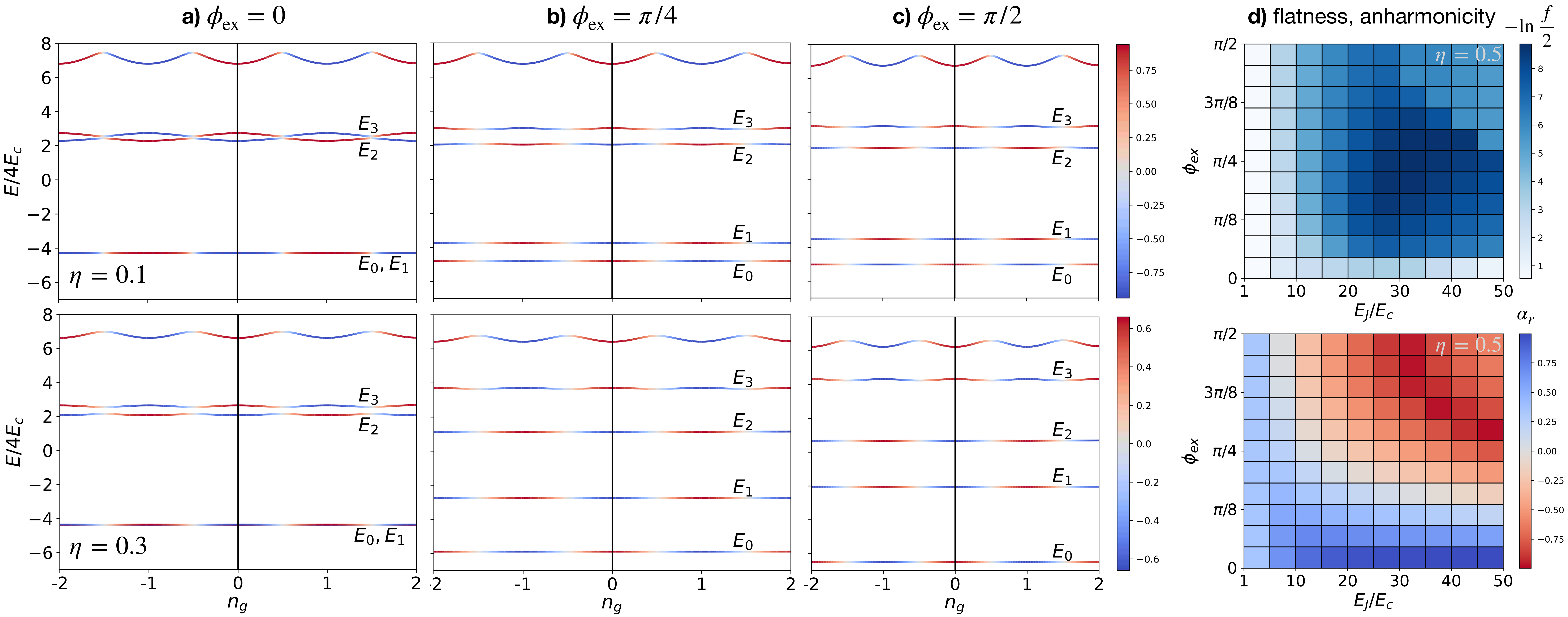
$$\phi_{\text{ex}}=2\pi\Phi/\Phi_0-\pi/2$$

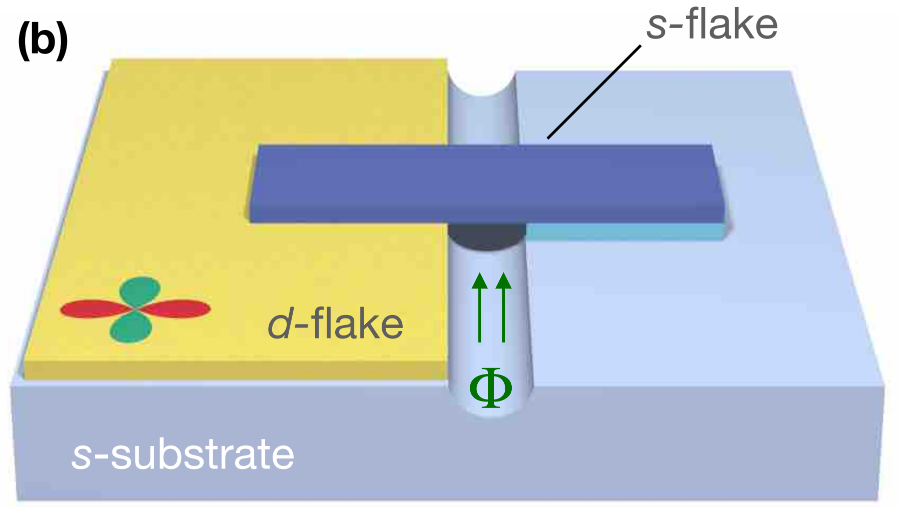
$$\eta=\frac{E_S}{E_J}\ll 1$$

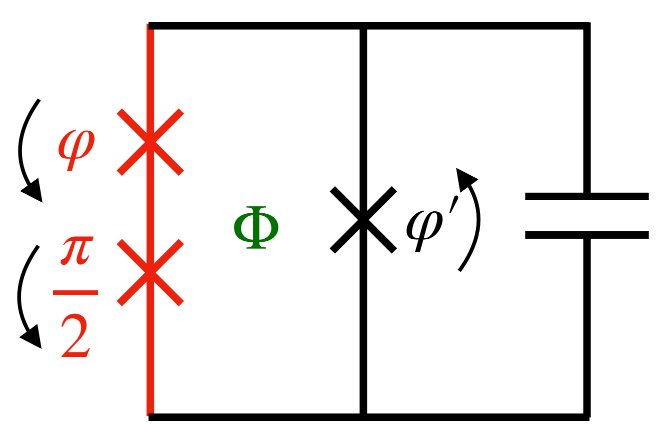
Split *d*-mon: a practical tunable qubit

[arXiv:2308.02547]

Split *d*-mon energy spectrum for $E_J/E_C = 32$







$$\phi_{\rm ex} = 2\pi\Phi/\Phi_0 - \pi/2, \quad \eta = \frac{E_S}{E_J}$$

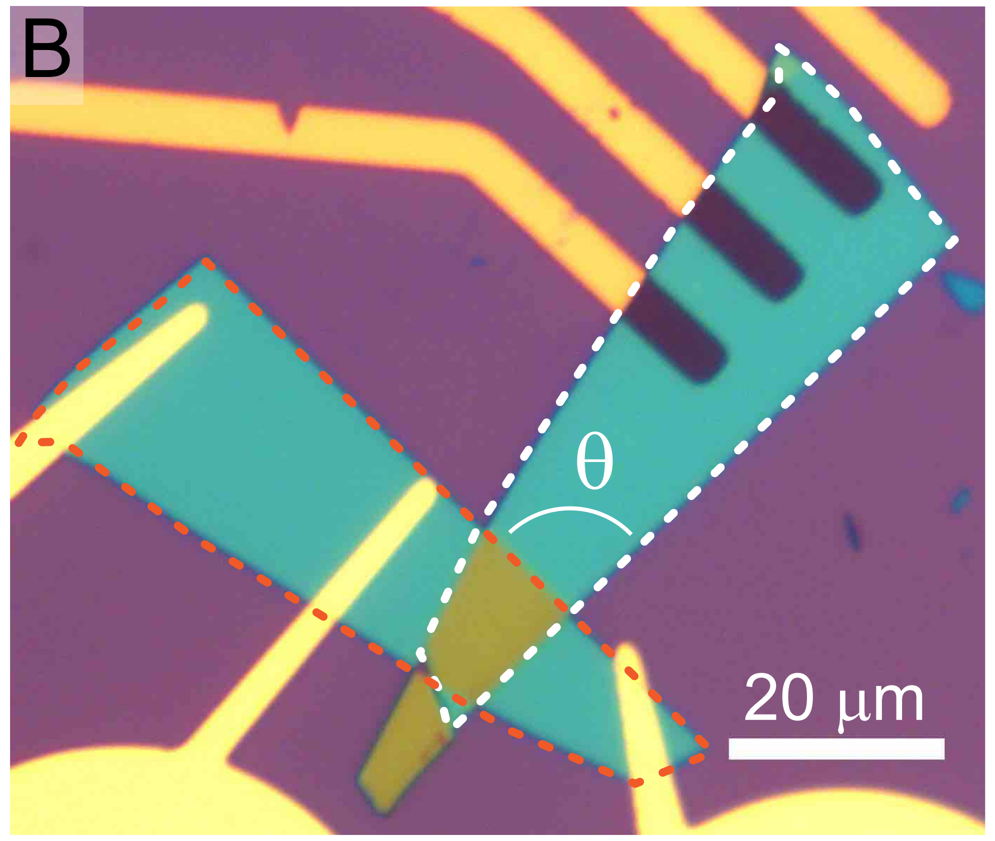
*Achieves similar insensitivity to offset charge but also **tunable and potentially large anharmonicity**.*

Split *d*-mon Schrodinger equation:

$$\left[4E_C\left(-i\frac{d}{d\varphi} - n_g\right)^2 - E_J\cos 2\varphi + E_S\cos(\varphi - \phi_{\rm ex})\right]\psi(\varphi) = E\psi(\varphi),$$

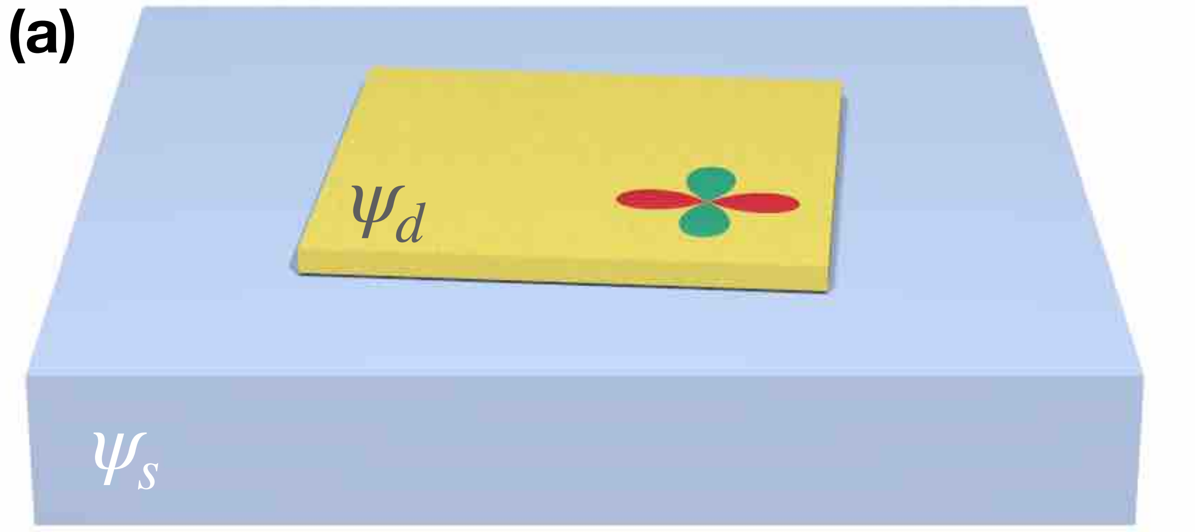
s/d junction versus twisted d/d

[arXiv:2308.02547]



Q: Can one use a twisted d/d junction to make a d-mon qubit?

A: In principle yes, but there is an issue of gapless quasiparticles in the bare dSC. Uncontrolled low-energy quasiparticles are detrimental for the qubit coherence, a.k.a. "quasiparticle poisoning".

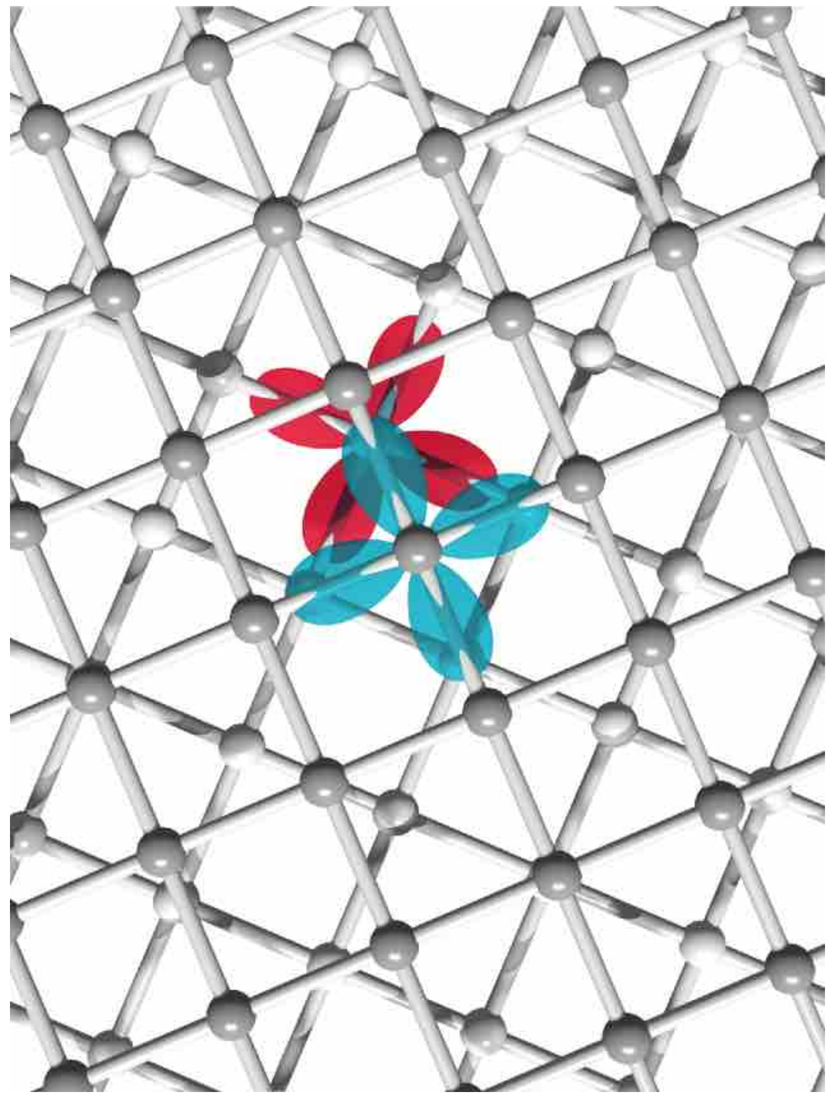


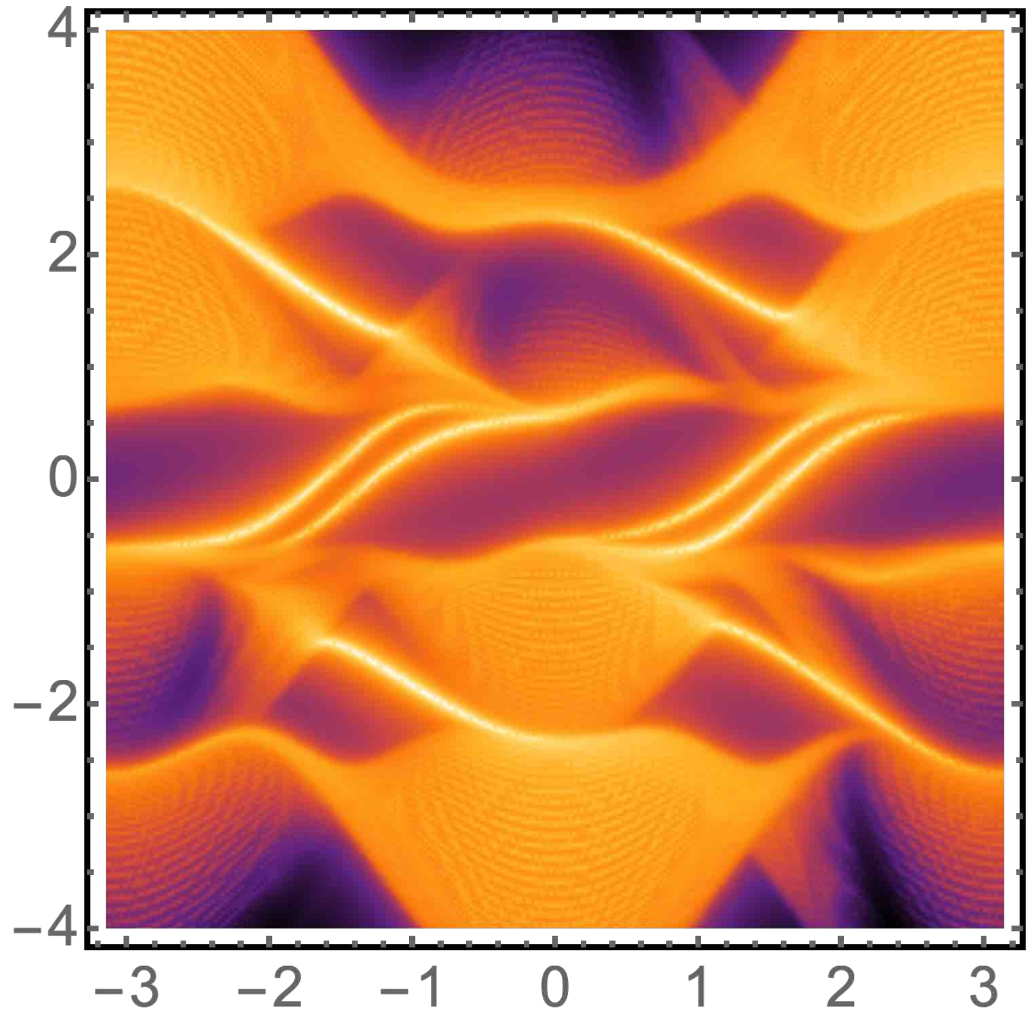
By contrast, for a thin dSC flake, the s-wave substrate induces a proximity gap, hence removing the issue of low-energy quasiparticles.

Remarkably, as we show in arXiv:2308.02547 this remains true even in the presence of a fluctuating relative phase — this is not apriori obvious and requires a subtle calculation.

Summary and outlook

- Natural models of coupled layers of *d*-wave SC predict a T-broken phase when the twist angle is close to 45°
- The resulting phase is fully gapped and over much of the phase diagram also topologically non-trivial
- Topological phase will show an even number of protected chiral edge modes
- Gap opening can be detected through various spectroscopies (ARPES, STM)
- T-breaking can be probed directly (polar Kerr effect, SC diode effect, fractional Shapiro steps)





Some interesting open questions:

1. What is the best way to observe the topological phase experimentally?
2. Are there any interesting uses for this novel topological superconducting phase once identified?
3. Are there other 2D systems (beyond graphene, chalcogenides, cuprates) that will produce interesting new behaviors under twist or similar geometries?