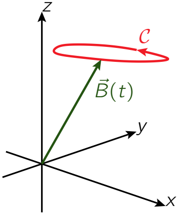
Floquet theory

Sigmund Kohler

Instituto de Ciencia de Materiales de Madrid, CSIC



I. Schrödinger equation

II. Quantum dissipation

III. Application: Landau-Zener-Stückelberg-Majorana interference

IV. Miscellaneous: symmetries, time-dep. Liouvillians, bichromatric, . . .

https://sigmundkohler.github.io/download/FloquetTutorial.pdf

- Time evolution of an eigenstate:

$$|\psi(t)\rangle = e^{-iE_n t}|\phi_n\rangle$$

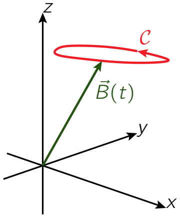
Notation:
ψ: solution of Schrödinger equation
ϕ: other state vector, e.g., eigenstate

- energy $\leftrightarrow$ phase

for (periodically) time-dependent system ?

- Spin in magnetic field
  $B(t) = B(t+T)$:

$$H(t) = \frac{1}{2}\vec{B}(t)\cdot\vec{\sigma}$$



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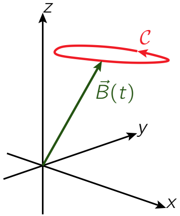
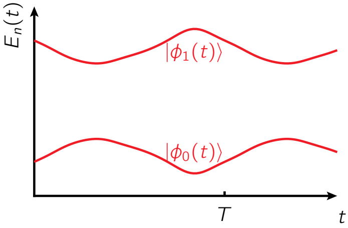


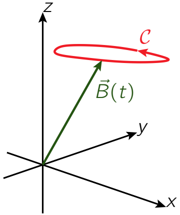
- Quantum dynamics for $\dot{B} \ll B^2$:
  state follows the eigenstate adiabatically

$$|\psi(t)\rangle \propto |\phi_n(t)\rangle$$



➔ $|\psi(t)\rangle$ determined up to phase factor

Berry phase



After one period: $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$

$$\varphi = -\int_0^T dt\, E_n(t) + \gamma_{\mathcal{C}}$$

- dynamical phase $\leftrightarrow$ mean energy
- Berry phase $\gamma_{\mathcal{C}}$
  - depends only on closed curve $\mathcal{C}$ in parameter space

M. Berry, Proc. Roy. Soc. London, Ser. A **392**, 45 (1984)

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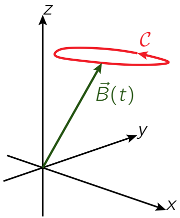

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- Assumptions:
  1. $\vec{B}(t)$ changes adiabatically slowly
  2. initial state: eigenstate $|\phi_n(0)\rangle$

Non-adiabatic generalization

Different perspective:

State vector undergoes periodic time-evolution

- $|\psi(T)\rangle = e^{i\varphi}|\psi(0)\rangle$
- dynamics $|\psi(t)\rangle$ induced by some Hamiltonian $H(t)$

Remarks:

- no adiabatic condition
- $|\psi(t)\rangle$ need not be an eigenstate of $H(t)$
- $H(t)$ is not unique

- only condition: cyclic time-evolution in Hilbert space

$$|\psi(t)\rangle = e^{if(t)}|\phi(t)\rangle, \qquad \phi(t) = \phi(t+T)$$

Non-adiabatic generalization

➔ Phase acquired during cyclic evolution: $\varphi = f(T) - f(0)$

$$\frac{df}{dt} = -\langle\phi|H|\phi\rangle + \langle\phi|i\frac{d}{dt}|\phi\rangle \quad \Longrightarrow \quad \boxed{\varphi = \gamma_{\mathrm{dyn}} + \gamma}$$

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Dynamical phase $\gamma_{\text{dyn}} = -\int_0^T dt\, \langle\phi(t)|H(t)|\phi(t)\rangle$

- depends on choice of $H(t)$
- reflects mean energy

Non-adiabatic generalization

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Aharonov-Anandan phase ("non-adiabatic Berry phase")

$$\gamma = \int_0^T dt\ \langle\phi|i\frac{d}{dt}|\phi\rangle$$

- depends only on trajectory in Hilbert space — not in parameter space!
- adiabatic limit: $\gamma = \gamma_C$

Aharonov & Anandan, PRL 1987

1. **Schrödinger equation**
   - Geometric phases
   - Time-periodicity, Floquet ansatz, and all that
2. **Quantum dissipation**
   - System-bath model
   - Floquet-Bloch-Redfield formalism
3. **Application: LZSM Interference**
4. **Miscellaneous**
   - Time-periodic Liouvillians
   - Symmetries
   - Bichromatic driving

Goal: propagator $U(t, t')$

- Time-independent system: diagonalize Hamiltonian ➔ $|\phi_n\rangle$, E_n

$$U(t, t') = U(t - t') = \sum_n e^{-iE_n(t-t')} |\phi_n\rangle\langle\phi_n|$$

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- Driven system:

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle$$ ➔ numerical integration

problem 1: time-integration not efficient for long times

problem 2: no information about structure of U

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- Solution for $H(t) = H(t + T)$: "Bloch theory in time"
  cf. $H(x)|\phi\rangle = \epsilon|\phi\rangle$ with $H(x) = H(x + a)$
  ➔ Bloch waves $\phi(x) = e^{iqx}\varphi(x)$, where $\varphi(x)$ is a-periodic

F. Bloch, Z. Phys. A **52**, 555 (1928)

Discrete time translation and Floquet ansatz

- $H(t) = H(t+T)$

  ➔ $t \to t+T$ is symmetry operation

  ➔ solutions of Schrödinger equation obey $|\psi(t+T)\rangle = e^{i\varphi}|\psi(t)\rangle$

- Floquet ansatz

$$|\psi(t)\rangle = e^{-i\epsilon t}|\phi(t)\rangle = e^{-i\epsilon t}\sum_k e^{-ik\Omega t}|c_k\rangle$$

  - ϵ quasienergy (cf. quasi momentum) ➔ long-time dynamics
  - $|\phi(t)\rangle = |\phi(t+T)\rangle$, Floquet state ➔ within driving period

- Floquet theorem: $H(t)$ has a complete set of Floquet solutions

- Schrödinger equation $i\partial_t|\psi\rangle = H(t)|\psi\rangle$ yields

$$\boxed{\big(H(t) - i\partial_t\big)|\phi(t)\rangle = \epsilon|\phi(t)\rangle}$$

Brillouin zone structure

- $|\phi(t)\rangle$ Floquet state with quasienergy ϵ

➔ $e^{ik\Omega t}|\phi(t)\rangle$ Floquet state with $\epsilon + k\Omega$

proof: insert into $(H - i\partial_t)|\phi\rangle = \epsilon|\phi\rangle$

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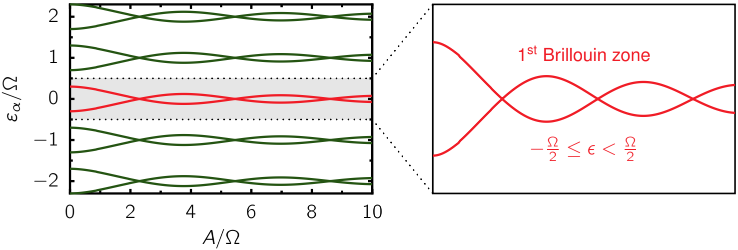
e.g. for two-level system



- all Brillouin zones equivalent, choice arbitrary

➔ quasienergies cannot serve for ordering!

- Physical quantity / observable: mean energy

$$E = \frac{1}{T}\int_0^T dt\ \langle\psi(t)|H(t)|\psi(t)\rangle = \frac{1}{T}\int_0^T dt\ \langle\phi(t)|H(t)|\phi(t)\rangle$$

- All equivalent states have the same mean energy
  [proof: insert $e^{-ik\Omega t}|\phi(t)\rangle$]

➔ Floquet states can be ordered by their mean energy

- Mean energy

$$E = \frac{1}{T}\int_0^T dt\, \langle\phi(t)|\{H(t) - i\partial_t + i\partial_t\}|\phi(t)\rangle$$

where $(H - i\partial_t)|\phi(t)\rangle = \epsilon|\phi(t)\rangle$

$$E = \epsilon + \frac{1}{T}\int_0^T dt\, \langle\phi(t)|i\partial_t|\phi(t)\rangle$$

- Mean energy

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$$-\epsilon = -E + \frac{1}{T}\int_0^T dt\, \langle \phi(t)|\, i\partial_t \,|\phi(t)\rangle$$

- Compare to $\varphi = \gamma_{\text{dyn}} + \gamma$

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- Compare to $\varphi = \gamma_{\mathrm{dyn}} + \gamma$

$(E - \epsilon)T$ is a geometric phase

Floquet equation as function of Ω and Ωt:

$$\epsilon(\Omega)\phi(\Omega t) = \left[H(\Omega t) - i\Omega\frac{\partial}{\partial \Omega t}\right]\phi(\Omega t)$$

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Compute derivative $(\partial/\partial\Omega)\big|_{\Omega t}$ and apply $\int \frac{dt}{T}\,\phi^{+}$ to obtain

$$\frac{\partial\epsilon}{\partial\Omega} = -\frac{\gamma}{2\pi} = \frac{\epsilon - E}{\Omega} \quad \rightarrow \quad \boxed{E = \epsilon - \Omega\frac{\partial\epsilon}{\partial\Omega}}$$

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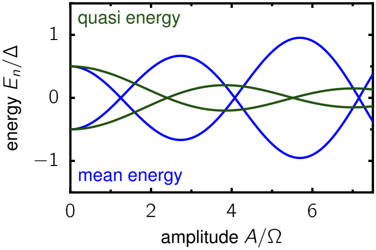
E.g. for $\gamma = \gamma_{\mathrm{ad}} + \mathcal{O}(\Omega)$

➔ $\epsilon = \mathrm{const} - (\gamma_{\mathrm{ad}}/2\pi)\Omega + \ldots$ ➔ $E = E_{\mathrm{ad}} + \mathcal{O}(\Omega^2)$

Fainshtein, Manakov, Rapoport, J. Phys. B (1978)

Mean energies — two-level system



Driven undetuned two-level system

- exact crossings
  (consequence of a symmetry)

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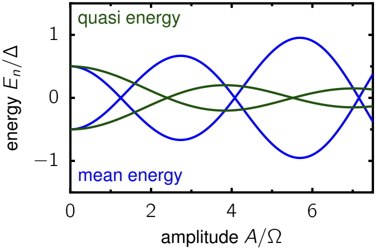

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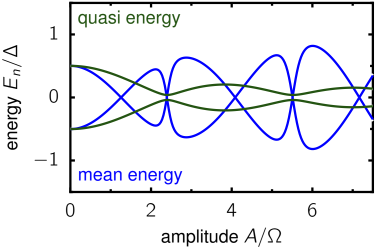

. . . with small detuning

- quasi energies
  - avoided crossings
- mean energies
  - exact crossings remain
  - additional crossings
  - ➔ do not follow from any eigenvalue equation

Goal: more formal treatment of $H(t) - i\partial_t$

- $|\phi(t)\rangle \in \mathcal{R} \otimes \mathcal{T}$ composite Hilbert space / Sambe space

  Shirley, PR **138**, B979 (1965), Sambe, PRA **7**, 2203 (1973)

  $\mathcal{T}$: Hilbert space of T-periodic functions with inner product

$$\langle f|g\rangle = \int_0^T f(t)^* \, g(t) \frac{dt}{T} = \sum_k f_k^* g_k$$

- extended Dirac notation:
  - $|\phi(t)\rangle = \langle t|\phi\rangle\rangle$
  - Fourier coefficient $|\phi_k\rangle = \langle k|\phi\rangle\rangle$

e.g.: $|\phi(t)\rangle = \langle t|\phi\rangle\rangle = \sum_k \langle t|k\rangle \langle k|\phi\rangle\rangle = \sum_k e^{-ik\Omega t}|\phi_k\rangle$

Completeness and Orthogonality

- $H - i\partial_t$ is hermitian

➔ Floquet states $|\phi_\alpha\rangle$ orthonormal and complete in $\mathcal{R} \otimes \mathcal{T}$

$$\langle\langle \phi_\alpha^{(k)} | \phi_\beta^{(k')} \rangle\rangle = \delta_{\alpha\beta}\delta_{kk'}$$

? but in $\mathcal{R}$?

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? but in $\mathcal{R}$?

- Consider $\langle \phi_\alpha(t) | \phi_\beta(t) \rangle = \sum_k \lambda_k e^{-ik\Omega t}$ since T-periodic

  with the Fourier coefficient

$$\lambda_k = \frac{1}{T} \int_0^T dt\, e^{ik\Omega t} \langle \phi_\alpha(t) | \phi_\beta(t) \rangle = \langle\langle \phi_\alpha | \phi_\beta^{(k)} \rangle\rangle = \delta_{\alpha\beta}\delta_{k,0}$$

→ Floquet states orthogonal at equal times

Propagator

- propagator in terms of Floquet states

$$U(t,t') = \sum_\alpha |\psi_\alpha(t)\rangle\langle\psi_\alpha(t')| = \sum_\alpha e^{-i\epsilon_\alpha(t-t')}|\phi_\alpha(t)\rangle\langle\phi_\alpha(t')|$$

  - long-time dynamics (depends on $t - t'$)
  - dynamics within driving period (depends on t and t')

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  - long-time dynamics (depends on $t - t'$)
  - dynamics within driving period (depends on t and t')

- one-period propagator for kicked systems

$$H(t) = H_0 + K\sum_n \delta(t - nT)$$

➔ $U(T) = e^{-iH_0 T} e^{-iK}$

✓ easy to compute
✓ provides quasienergies
✗ only long-time dynamics (stroboscopic)

Solve eigenvalue problem

$$\left\{H(t) - i\partial_t\right\}|\phi\rangle\rangle = \epsilon|\phi\rangle\rangle$$

Computation of Floquet states

Solve eigenvalue problem

$$\left\{H(t) - i\partial_t\right\}|\phi\rangle\rangle = \epsilon|\phi\rangle\rangle$$

Straightforward in Fourier representation (“Floquet matrix”)

$$H_0 + H_1\cos(\Omega t) - i\frac{d}{dt} \leftrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & H_0 + 2\Omega & \frac{1}{2}H_1 & 0 & 0 & 0 & \cdots \\ \cdots & \frac{1}{2}H_1 & H_0 + \Omega & \frac{1}{2}H_1 & 0 & 0 & \cdots \\ \cdots & 0 & \frac{1}{2}H_1 & H_0 & \frac{1}{2}H_1 & 0 & \cdots \\ \cdots & 0 & 0 & \frac{1}{2}H_1 & H_0 - \Omega & \frac{1}{2}H_1 & \cdots \\ \cdots & 0 & 0 & 0 & \frac{1}{2}H_1 & H_0 - 2\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

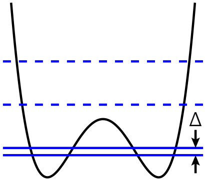
Computation of Floquet states

1. direct diagonalization of $H(t) - i\partial_t$
   - conceptually simple ➔ first choice
   - increasingly difficult with smaller frequency
   - often more efficient after unitary transformation
2. analytical tool: perturbation theory
   strong driving: $H_1 \cos(\Omega t) - i\partial_t$ as zeroth order

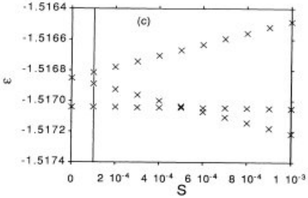
3. diagonalization of $U(T, 0)$ ➔ $e^{-i\epsilon T}$, $|\phi(0)\rangle$
4. matrix-continued fraction (convenient for time-dep. Liouvillians)
5. (t, t') formalism (very efficient propagation scheme)

1. role of quasienergy crossings
2. perturbation theory (two-level approximation)

Driven double-well potential $H(t) = H_{\text{DW}} + Sx\cos(\Omega t)$



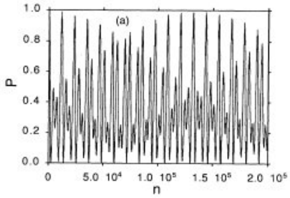
quasienergies $\epsilon_\alpha(S)$

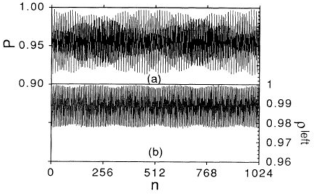


? tunnel oscillations influenced by driving

? dynamics at quasienergy crossing

Occupation $P_{\text{left}}(nT)$





far from crossing:

- tunnel oscillations

at crossing:

- particle stays in left well

➔ "coherent destruction of tunneling" by ac field

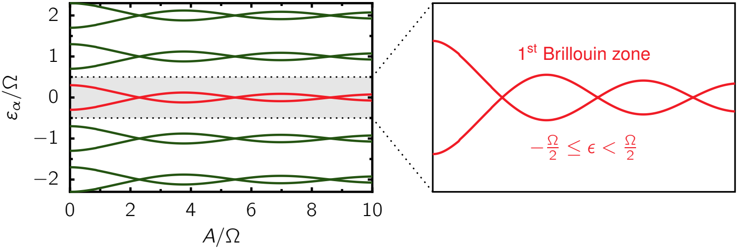
Grossmann *et al.*, PRL 1991

Analytical understanding ➔ two-level approximation

Driven two-level system

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

quasienergy spectrum



Analytical approach for $\Delta \ll \Omega$: high-frequency limit

$$H(t) = -\frac{\Delta}{2}\sigma_x + \frac{A}{2}\cos(\Omega t)\sigma_z$$

- zeroth order propagator

$$U_{\mathrm{dr}}(t,0) = \exp\left(-\frac{iA}{2\Omega}\sin(\Omega t)\sigma_z\right)$$

- transformation and rotating-wave approximation (i.e. time-average)

$$H(t) \longrightarrow -\frac{\Delta}{2}U_{\mathrm{dr}}^{\dagger}(t,0)\sigma_x U_{\mathrm{dr}}(t,0) \longrightarrow -\frac{\Delta}{2}J_0(A/\Omega)\sigma_x$$

➔ tunnel-matrix element renormalized by Bessel function J_0

Bessel function $J_n(x)$: nth Fourier coefficent of $e^{-ix\sin(\Omega t)}$

Example I: CDT — perturbation theory

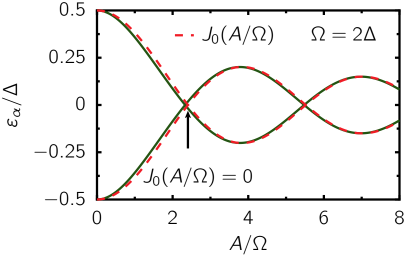
$$H_{\text{eff}} = \frac{\tilde{\Delta}}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- Floquet states

$$\phi_{\pm}(t) = U_{\text{dr}}(t,0)|\pm\rangle$$

- quasienergies

$$\epsilon_{\pm} = \pm\frac{\Delta}{2}J_0(A/\Omega)$$



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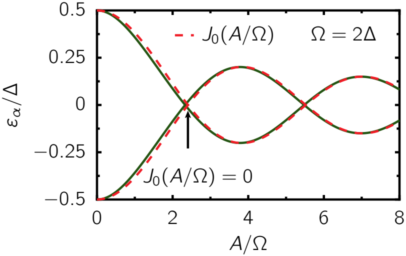

- mean energies $E = \epsilon - \Omega(\partial\epsilon/\partial\Omega)$

$$E_\pm = \pm\frac{\Delta}{2}\left[J_0(A/\Omega) - \frac{A}{\Omega}J_1(A/\Omega)\right]$$

Bessel function $J_1(x) = \frac{d}{dx}J_0(x)$

Some standard references

- Classic work:
  - Shirley, Phys. Rev. 138, B979 (1965)
  - Sambe, Phys. Rev. A 7, 2203 (1973)
- Reviews:
  - Grifoni, Hänggi, Phys. Rep. 304, 229 (1998)
  - Hänggi, Chap.5 of “Quantum transport and dissipation” (1998)
    http://www.physik.uni-augsburg.de/theo1/hanggi/Papers/Chapter5.pdf
  - Eckardt, Rev. Mod. Phys. 89, 011004 (2017)

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Heuristic approach

coupling of qubit to electromagnetic environment ➔ sponaneous decay

$$|\psi\rangle \longrightarrow \begin{cases} \sigma_-|\psi\rangle & \text{decay with probability } \alpha \ll 1 \\ |\psi\rangle + |\delta\psi\rangle & \text{no decay, probability } 1-\alpha \end{cases}$$

- normalization requires $|\delta\psi\rangle = \frac{\alpha}{2}\sigma_+\sigma_-|\psi\rangle$

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- normalization requires $|\delta\psi\rangle = \frac{\alpha}{2}\sigma_+\sigma_-|\psi\rangle$
- corresponding density operator

$$\rho \longrightarrow \rho + \frac{\alpha}{2}\Big(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\Big)$$

- add continuous time-evolution ➔ master equation

$$\frac{d}{dt}\rho = -i[H,\rho] + \frac{\gamma}{2}\Big(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\Big)$$

Lindblad form

Time evolution must conserve

- hermiticity and trace of ρ
- positivity (all eigenvalues of $\rho \geq 0$)

Lindblad form

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- positivity (all eigenvalues of $\rho \geq 0$)

Fulfilled by a Markovian master equation iff of "Lindblad form"

$$\frac{d}{dt}\rho = -i[H,\rho] + \sum_n \gamma_n \left(2Q_n\rho Q_n^\dagger - Q_n^\dagger Q_n\rho - \rho Q_n^\dagger Q_n\right)$$

G. Lindblad, Comm. Math. Phys. **48**, 119 (1976)

V. Gorini, J. Math. Phys. **17**, 821 (1976)

- Interpretation: incoherent transitions $|\psi\rangle \rightarrow Q_n|\psi\rangle$

Lindblad form

Time evolution must conserve

- hermiticity and trace of ρ
- positivity (all eigenvalues of $\rho \geq 0$)

Fulfilled by a Markovian master equation iff of "Lindblad form"

$$\frac{d}{dt}\rho = -i[H,\rho] + \sum_n \gamma_n \Big(2Q_n\rho Q_n^\dagger - Q_n^\dagger Q_n \rho - \rho Q_n^\dagger Q_n \Big)$$

G. Lindblad, Comm. Math. Phys. **48**, 119 (1976)

V. Gorini, J. Math. Phys. **17**, 821 (1976)

- Interpretation: incoherent transitions $|\psi\rangle \to Q_n|\psi\rangle$

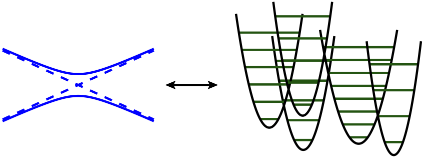
✗ Critique

- request for Markovian evolution unphysical
- axiomatic, not based on physical model
- high-temperature limit typically wrong
  i.e. not the Klein-Kramers or the Smoluchowski equation

Caldeira-Leggett model

Magalinskii 1959; Caldeira, Leggett 1981

Coupling of a system to bath of harmonic oscillators



$$H = H_{\text{system}}(t) + X \sum_{\nu} \gamma_{\nu} (b_{\nu}^{\dagger} + b_{\nu}) + \sum_{\nu} \omega_{\nu} b_{\nu}^{\dagger} b_{\nu}$$

→ eliminate bath

→ equation of motion for reduced density operator

- interpretation: bath “measures” system operator X

Total density operator $R \approx \rho \otimes \rho_{\text{bath,eq}}$

$$\dot{R} = -i[H_{\text{total}}, R]$$

2nd order perturbation theory in system-bath coupling

$$\frac{d}{dt}\rho = -i[H_{\text{sys}}, \rho] - i\int_0^{(t-t_0)\to\infty} d\tau \mathcal{A}(\tau)[X, [\tilde{X}(-\tau), \rho(t-\tau)]_+] - \int_0^{(t-t_0)\to\infty} d\tau \mathcal{S}(\tau)[X, [\tilde{X}(-\tau), \rho(t-\tau)]]$$

- Heisenberg operator $\tilde{X}(-\tau) = U(\tau)\, X\, U^\dagger(\tau)$
- bath correlation functions $\mathcal{A}$, $\mathcal{S}$
- non-Markovian
- short system-bath correlation time: Markov approximation

- anti-symmetric correlation function

$$\mathcal{A}(\tau) = -i\langle[\xi(\tau), \xi(0)]\rangle$$

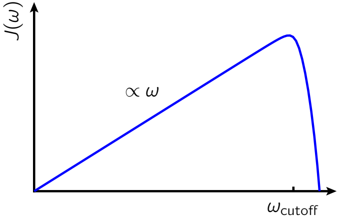
- Fourier transformed: spectral density $\longrightarrow$ continuum limit

$$\mathcal{A}(\omega) = \pi \sum_{\nu} |\gamma_\nu|^2 \delta(\omega - \omega_\nu) \longrightarrow J(\omega)$$

- here: Ohmic with cutoff

$$J(\omega) = 2\pi\alpha\omega e^{-\omega/\omega_{\mathrm{cutoff}}}$$

- dimensionless dissipation strength α

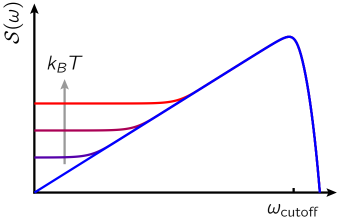


- symmetric bath correlation function

$$\mathcal{S}(\tau) = \frac{1}{2}\langle[\xi(\tau),\xi(0)]_+\rangle$$

$$\mathcal{S}(\omega) = J(\omega)\coth\left(\frac{\omega}{2k_BT}\right)$$

$$= \begin{cases} 4\pi\alpha k_{\mathrm{B}}T & \text{high } k_BT \\ 2\pi\alpha\omega & \text{low } k_BT \end{cases}$$



Bath correlation functions

- symmetric bath correlation function

$$\mathcal{S}(\tau) = \frac{1}{2}\langle[\xi(\tau),\xi(0)]_+\rangle$$

$$\mathcal{S}(\omega) = J(\omega)\coth\left(\frac{\omega}{2k_BT}\right)$$

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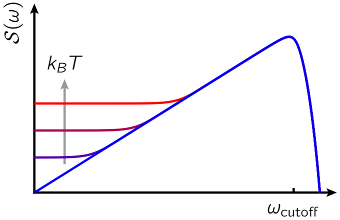

- $\mathcal{S}(\omega)$ evaluated at transition frequencies

→ dissipation strength depends on coherent spectrum/dynamics

- Ohmic, short memory times (e.g. for $\gamma < k_{\mathrm{B}} T$)
  ➔ Bloch-Redfield master equation

$$\dot{\rho} = -i[H_{\mathrm{S}}, \rho] + i\gamma[X, \{[H_{\mathrm{S}}, X], \rho\}] - [X, [Q, \rho]]$$

coherent dynamics   dissipation   decoherence

coherent dynamics enters via $Q = \int_0^\infty d\tau\, \mathcal{S}(\tau)\, \tilde{X}(-\tau)$

- Ohmic, short memory times (e.g. for $\gamma < k_\mathrm{B} T$)
  ➔ Bloch-Redfield master equation

$$\dot{\rho} = -i[H_\mathrm{S}, \rho] + i\gamma[X, \{[H_\mathrm{S}, X], \rho\}] - [X, [Q, \rho]]$$

coherent dynamics   dissipation   decoherence

coherent dynamics enters via $Q = \int_0^\infty d\tau\, \mathcal{S}(\tau)\, \tilde{X}(-\tau)$

- not of Lindblad form
  - ✗ positivity might be violated
  - ✓ happens only on unphysically small time scales
- high-temperature limit: Fokker-Planck equation

- Decomposition into energy basis and rotating-wave approximation

➔ rate equation for the populations (Pauli master equation)

$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\alpha'} \left[w_{\alpha\leftarrow\alpha'}\,\rho_{\alpha'\alpha'} - w_{\alpha'\leftarrow\alpha}\,\rho_{\alpha\alpha} \right]$$

with the golden-rule rates

$$w_{\alpha\leftarrow\alpha'} = J(E_\alpha - E_{\alpha'})\,\left|\langle\phi_\alpha|X|\phi_{\alpha'}\rangle\right|^2\, n_{\rm th}(E_\alpha - E_{\alpha'})$$

- notice: $-n_{\rm th}(-\omega) = n_{\rm th}(\omega)+1$

icmm

- Decomposition into energy basis and rotating-wave approximation

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- notice: $-n_{\text{th}}(-\omega) = n_{\text{th}}(\omega) + 1$

✓ fluctuation theorem $\frac{w_{\alpha\leftarrow\alpha'}}{w_{\alpha'\leftarrow\alpha}} = e^{-(E_\alpha - E_{\alpha'})/k_B T}$

✓ Lindblad form

✗ high-temperature limit typically wrong

full Bloch-Redfield: golden rule for non-diagonal $\rho_{\alpha\beta}$

Driven system ➔ noise term becomes time-dependent

$$\dot{\rho} = \ldots - [X, [Q(t), \rho]], \quad Q(t) = \int_0^\infty d\tau \, \mathcal{S}(\tau) \, \tilde{X}(t-\tau, t)$$

Driven system ➔ noise term becomes time-dependent

$$\dot{\rho} = \ldots - [X, [Q(t), \rho]], \quad Q(t) = \int_0^\infty d\tau \, \mathcal{S}(\tau) \, \tilde{X}(t - \tau, t)$$

Central idea:

1. adapted basis: Floquet states $|\phi_\alpha(t)\rangle$ ➔ captures coherent dynamics
2. master equation in Floquet basis

$$\frac{d}{dt}\rho_{\alpha\beta} = -i(\epsilon_\alpha - \epsilon_\beta)\rho_{\alpha\beta} + \sum_{\alpha'\beta'} \mathcal{L}_{\alpha\beta,\alpha'\beta'}(t) \, \rho_{\alpha'\beta'}$$

   where $\mathcal{L}(t) = \mathcal{L}(t + T)$

3. moderate rotating-wave approximation:
   time average $\mathcal{L}(t) \to \bar{\mathcal{L}}$, but keep all $\rho_{\alpha\beta}$

Blümel *et al.*, PRA 1991; SK, Dittrich, Hänggi, PRE 1997

Floquet-Bloch-Redfield master equation — recipe

- Numerical method: compute $\mathcal{L}$ and solve

$$\dot{\rho}_{\alpha\beta} = -i(\epsilon_\alpha - \epsilon_\beta)\rho_{\alpha\beta} + \sum_{\alpha'\beta'} \bar{\mathcal{L}}_{\alpha\beta,\alpha'\beta'}\, \rho_{\alpha'\beta'}$$

  1. time-independent master equation for driven system
  2. ac driving captured by choice of basis ➔ efficient
  3. includes impact of bath on dissipation strength (very relevant for fermionic baths; see Lecture III)

- Analytical tool: find H_{eff} and approx. for $\overline{Q(t)}$

  ➔ effective time-independent Bloch-Redfield equation

➔ full RWA ➔ (Pauli master equation)

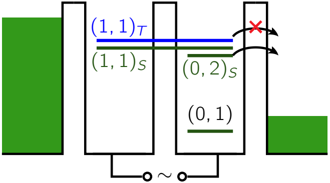
$$\frac{d}{dt}\rho_{\alpha\alpha} = \sum_{\alpha'} w_{\alpha\leftarrow\alpha'}\,\rho_{\alpha'\alpha'} - \sum_{\alpha} w_{\alpha'\leftarrow\alpha}\,\rho_{\alpha\alpha}$$

with the golden-rule rates

$$w_{\alpha\leftarrow\alpha'} = \sum_k J(\epsilon_\alpha - \epsilon_{\alpha'} + k\Omega)\left|\sum_{k'}\langle\phi_{\alpha,k+k'}|X|\phi_{\alpha',k}\rangle\right|^2 n_{\mathrm{th}}(\epsilon_\alpha - \epsilon_{\alpha'} + k\Omega)$$

- sidebands contribute to $w_{\alpha\leftarrow\alpha'}$

  ... but NOT as independent states!
- no simple relation between forward/backward rates



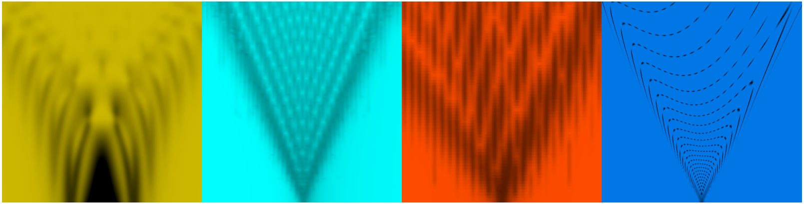
- Floquet states for central system
- evaluate rates $w_{\alpha\leftarrow\alpha'}$

→ dc current, counting statistics

	Dissipation	Transport
Environment	harmonic oscillators	electron source/drain
Coupling of mode ν	$X(a_\nu^\dagger + a_\nu)$	$c^\dagger c_\nu + c_\nu^\dagger c$
Absorption / tunnel in	$n_{\text{th}}(\omega)$	$f(\epsilon - \mu)$
Emission / tunnel out	$1 + n_{\text{th}}(\omega)$	$1 - f(\epsilon - \mu)$
“Ohmic”	$J(\omega) \propto \omega$	$\Gamma(\omega) = \text{const}$

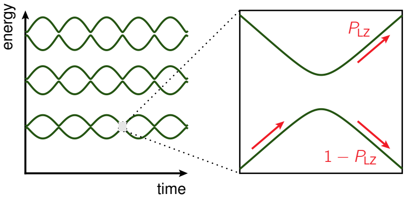
1. **Schrödinger equation**
   - Geometric phases
   - Time-periodicity, Floquet ansatz, and all that
2. **Quantum dissipation**
   - System-bath model
   - Floquet-Bloch-Redfield formalism
3. **Application: LZSM Interference**
4. **Miscellaneous**
   - Time-periodic Liouvillians
   - Symmetries
   - Bichromatic driving

Landau-Zener-Stückelberg-Majorana Interference



Experiments by Gang Cao (Hefei), Stefan Ludwig (PDI Berlin), and Jason Petta (UCLA)

Quantum system in AC-field, $H(t)$



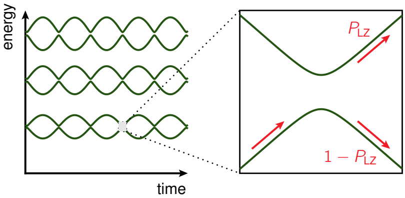
- non-adiabatic transition probability

$$P_{\mathrm{LZ}} = e^{-\pi\Delta^2/2\hbar v}$$

Landau, Zener, Stückelberg, Majorana, 1932

AC-driving and Landau-Zener transitions

Quantum system in AC-field, $H(t)$



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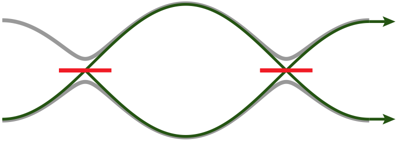


➔ beam splitter, interference

➔ Landau-Zener-(Stückelberg-Majorana) interferometry

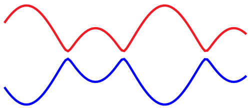
$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \qquad g(t) = \epsilon + A\cos(\Omega t)$$

1. (avoided) crossing requires $A > |\epsilon|$

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \qquad g(t) = \epsilon + A\cos(\Omega t)$$

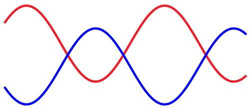
1. (avoided) crossing requires $A > |\epsilon|$
2. relative phase between dominant paths

adiabatic: $P_{\rm LZ} \ll 1$



$$\varphi(T) = \int_0^T dt\ |g(t)|$$

diabatic: $1 - P_{\rm LZ} \ll 1$



$$\varphi(T) = \int_0^T dt\ g(t)$$

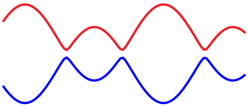
➔ fringes for $\varphi(T) = 2\pi k$

Interference condition

$$H(t) = \frac{\Delta}{2}\sigma_x + \frac{g(t)}{2}\sigma_z \qquad g(t) = \epsilon + A\cos(\Omega t)$$

1. (avoided) crossing requires $A > |\epsilon|$
2. relative phase between dominant paths

adiabatic: $P_{\mathrm{LZ}} \ll 1$



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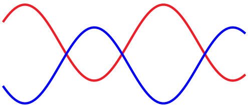
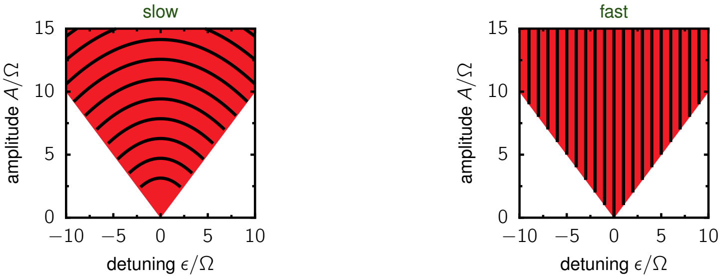


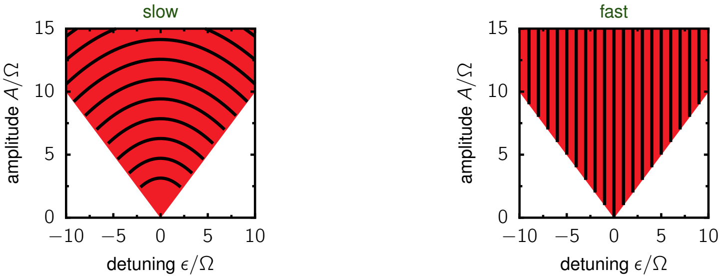
$$\varphi(T) = \int_0^T dt\, g(t) = \epsilon T$$

$\epsilon = k\Omega$ “k-photon resonance”

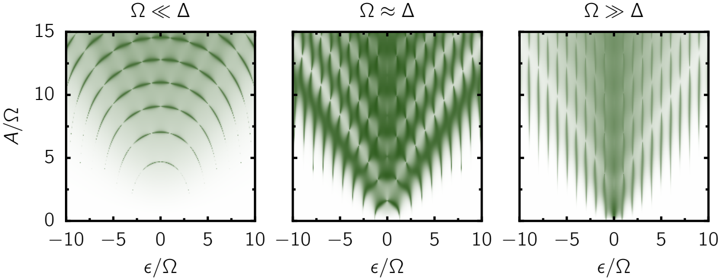
Patterns for two-level systems



Patterns for two-level systems

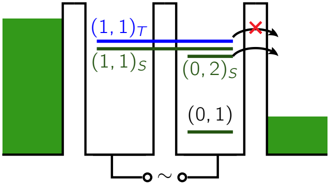


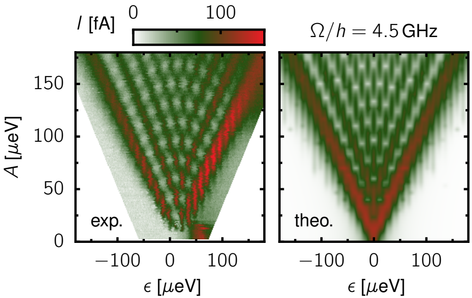
E.g. excitation probabilities of a TLS:



DC current:

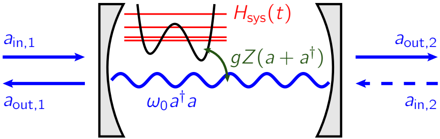
$$I(t) = e_0 \Gamma_R \langle n_R(t) \rangle \quad \rightarrow \quad \overline{\langle n_R(t) \rangle}^T$$





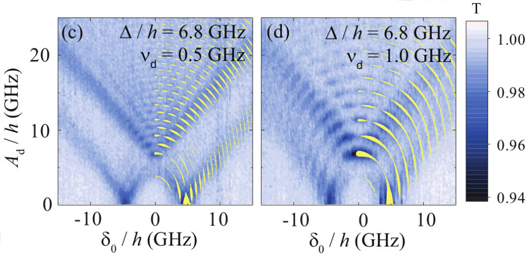
Forster *et al.*, PRL 2014

Measurement II: Cavity transmission



Dispersive frame:
effective cavity frequency

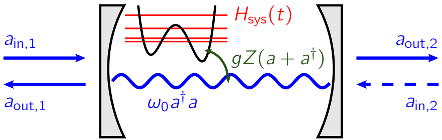
$$\omega_0 \longrightarrow \omega_0 + \frac{g^2}{\epsilon_{qb} - \omega_0} \sigma_z$$



Theory for readout of driven system?

Koski *et al.*, PRL 2018

Qubit-cavity Hamiltonian



$$H = H_{sys}(t) + gZ(a^\dagger + a) + \omega_0 a^\dagger a$$

- Backaction: cavity ➔ qubit ➔ cavity
- Cavity equation (input/output formalism)

$$\frac{d}{dt}a = -i\omega_0 a - \frac{\kappa}{2}a - \sum_{\nu=1,2} \sqrt{\kappa_\nu}\, a_{in,\nu} - igZ$$

- (non equilibrium) Kubo formula $Z(t) = g \int dt' \, \chi(t - t') a(t')$
  with the response function (may depend on the initial state!)

$$\chi(t) = -i\langle [Z(t), Z] \rangle \theta(t - t')$$

➔ $-i\omega a = -i\big(\omega_0 + g^2\chi(\omega)\big) a - \frac{\kappa}{2} a - \sum_{\nu=1,2} \sqrt{\kappa_\nu} a_{\mathrm{in},\nu}$

➔ measured quantity: (non-equilibrium) susceptibility

Response function for driven system

Response of periodically driven system

$$\chi(t, t') = -i\langle [Z(t), Z(t')] \rangle_{\text{non-eq}} = \chi(t+T, t'+T)$$

such that

$$\chi(t, t-\tau) = \sum_k e^{-ik\Omega t} \int d\omega\, e^{-i\omega\tau} \chi^{(k)}(\omega)$$

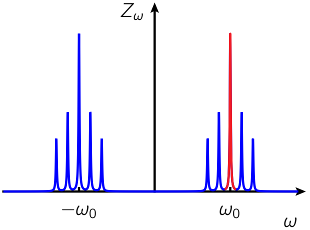
Response of periodically driven system

$$\chi(t,t') = -i\langle[Z(t),Z(t')]\rangle_{\text{non-eq}} = \chi(t+T,t'+T)$$

such that

$$\chi(t,t-\tau) = \sum_k e^{-ik\Omega t}\int d\omega\, e^{-i\omega\tau}\chi^{(k)}(\omega)$$

- resonant cavity driving, $\omega = \omega_0$
- response $Z(t)$ acquires sidebands
- good cavity limit, $\kappa \ll \omega_0, \Omega$



Response function for driven system

Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta,\alpha,k} \frac{(p_\alpha - p_\beta)|Z_{\beta\alpha,k}|^2}{\epsilon_\alpha - \epsilon_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory ➔ quasi-energies ϵ_α
- Floquet-Bloch-Redfield ➔ populations p_α

Response function for driven system

Relevant component:

$$\chi^{(0)}(\omega_0) = \sum_{\beta,\alpha,k} \frac{(p_\alpha - p_\beta)|Z_{\beta\alpha,k}|^2}{\epsilon_\alpha - \epsilon_\beta + \omega_0 + k\Omega + i\gamma/2}$$

- Floquet theory ➔ quasi-energies ϵ_α
- Floquet-Bloch-Redfield ➔ populations p_α

Resonance conditions

cavity response:

$$\Delta\epsilon = \omega_0 + k\Omega$$

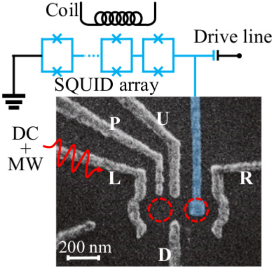
cf. population:

$$\Delta\epsilon = k\Omega$$

e.g. Ivakhnenko *et al.*, Phys.Rep. 2023

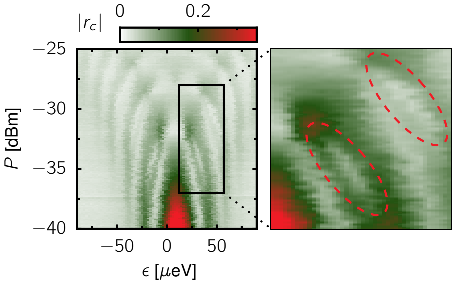
➔ Agree only for low-frequency oscillator!

Readout of Floquet state population



Chen *et al.*, Phys. Rev. B **103**, 205428 (2021)

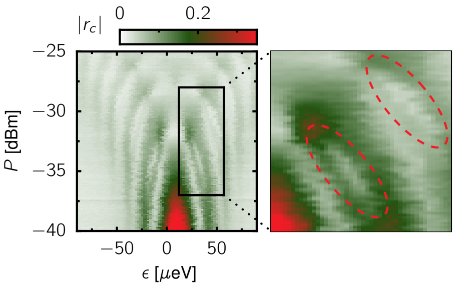
Motivation: Holes in interference fringes



Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD ➔ two-level sys.

Motivation: Holes in interference fringes



Experiment (Cao & Guo, Hefei)

- holes in LZSM pattern
- GaAs DQD ➔ two-level sys.

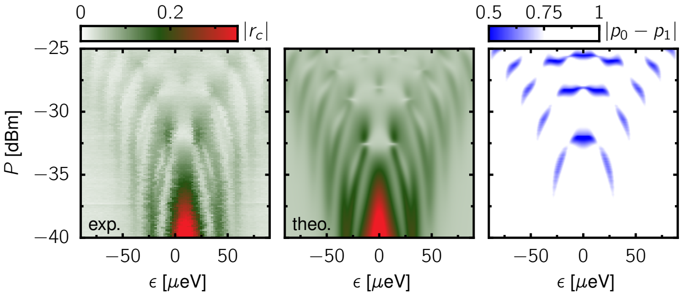
Recap: susceptibility (two-level system)

$$\chi^{(0)}(\omega_0) = (p_0 - p_1) \sum_k \frac{|Z_{10,k}|^2}{\epsilon_1 - \epsilon_0 + \omega_0 + k\Omega + i\gamma/2}$$

response determined by

- resonance condition for cavity signal
- Floquet state population

Experiment *vs.* Theory



- competing resonance conditions
- holes in fringes when $p_0 \approx p_1 \approx 1/2$

➔ cavity response provides information about Floquet state population

Mean energy state *vs.* Floquet-Gibbs state

Steady state of driven dissipative quantum system

quasi energy: $p_\alpha \propto e^{-\epsilon_\alpha/kT}$

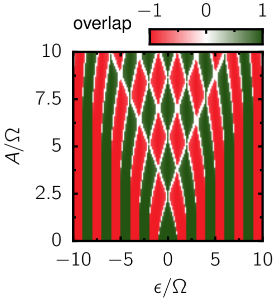
mean energy: $p_\alpha \propto e^{-E_\alpha/kT}$

Steady state of driven dissipative quantum system

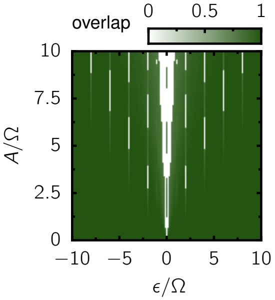
quasi energy: $p_\alpha \propto e^{-\epsilon_\alpha/kT}$

mean energy: $p_\alpha \propto e^{-E_\alpha/kT}$

bath via σ_z



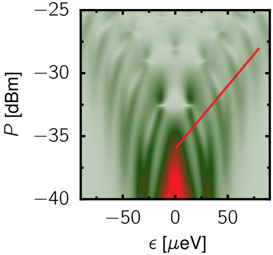
bath via σ_x

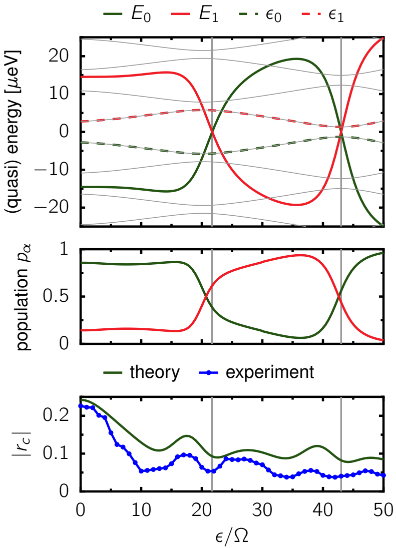


Floquet-Gibbs state
vs. anti Floquet-Gibbs

The present case!

Floquet state population





- p_α determined by E_α

→ holes in reflection consistent with mean-energy state

→ bath coupling (predominantly) via σ_x

1. **Schrödinger equation**
   - Geometric phases
   - Time-periodicity, Floquet ansatz, and all that
2. **Quantum dissipation**
   - System-bath model
   - Floquet-Bloch-Redfield formalism
3. **Application: LZSM Interference**
4. **Miscellaneous**
   - Time-periodic Liouvillians
   - Symmetries
   - Bichromatic driving

Master equation of type

$$\frac{d}{dt}P = L(t)P$$

- Floquet-Bloch-Redfield beyond moderate RWA
- time-dependent system with Lindblad dissipator
  $\dot{\rho} = -i[H(t), \rho] + \gamma(2a^\dagger \rho a - a^\dagger a\rho - \rho a^\dagger a)$
  - very weak dissipation
  - transport problem with large bias

→ long-time solution T-periodic

→ Floquet ansatz with "quasienergy" zero

$$P(t) = \sum_k e^{-ik\Omega t} p_k$$

Matrix-continued fractions

$\frac{d}{dt}P = L(t)P$ with

$$L(t) = L_0 + 2L_1 \cos(\Omega t)$$

➔ kernel of tridiagonal Floquet matrix

$$L_0 + 2L_1\cos(\Omega t) - \partial_t \leftrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & L_0 + 2i\Omega & L_1 & 0 & 0 & 0 & \cdots \\ \cdots & L_1 & L_0 + i\Omega & L_1 & 0 & 0 & \cdots \\ \cdots & 0 & L_1 & L_0 & L_1 & 0 & \cdots \\ \cdots & 0 & 0 & L_1 & L_0 - i\Omega & L_1 & \cdots \\ \cdots & 0 & 0 & 0 & L_1 & L_0 - 2i\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Matrix-continued fractions

- ansatz $P(t) = \sum_k e^{-ik\Omega t} p_k$ yields

$$L_1 p_{k-1} + (L_0 + ik\Omega) p_k + L_1 p_{k+1} = 0$$

- idea: truncate and iterate $p_{k-1} = -L_1^{-1} \left\{ (L_0 - ik\Omega) p_k + L_1 p_{k+1} \right\}$

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  - ✗ fails, L_1 generally singular
- solution: ansatz $p_k = S_k L_1 p_{k\mp 1}$ ($k \gtrless 0$) leads to

$$S_k = -\left(L_0 + ik\Omega + L_1 S_{k\pm 1} L_1\right)^{-1} \longrightarrow S_{\pm 1} \quad (1)$$

$$0 = \left(L_1 S_{-1} L_1 + L_0 + L_1 S_1 L_1\right) p_0 \quad (2)$$

➔ truncate at $\pm k_0$, iterate (1), and solve (2)

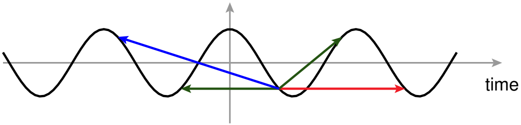
➔ time-averaged $P(t) = p_0$ ➔ time-averaged expectation values

Risken, “The Fokker-Planck Equation”

Appendix of Forster *et al.*, PRB 2015

Symmetries of dipole driving

$H_{\text{dipole}} \propto x \cos(\Omega t)$



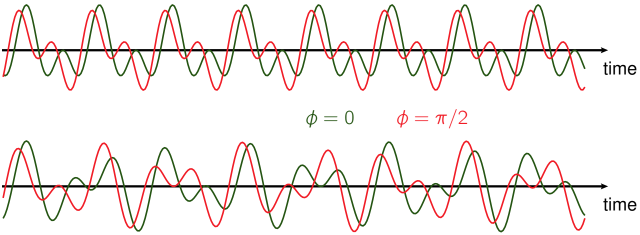
1. time periodicity $t \longrightarrow t + T$ ➔ Floquet theory applicable
2. time reversal $t \longrightarrow -t$ ➔ Floquet states real
3. generalized parity $(x, t) \longrightarrow (-x, t + T/2)$
   ➔ Floquet states even/odd
   e.g. symmetric potential with dipole driving
4. time-reversal parity $(x, t - T/4) \longrightarrow (-x, T/4 - t)$
   - combination of the other three
   - relevant for Floquet scattering theory

Bichromatic driving

- $g(t) = \cos(\Omega t) + \eta \cos(\Omega' t + \phi)$

  Ω', Ω commensurable *vs.* incommensurable

  ➔ $g(t)$ periodic *vs.* quasi-periodic



$\frac{\Omega'}{\Omega} = 2$

$\frac{\Omega'}{\Omega} = \frac{1+\sqrt{5}}{2}$

Two-color Floquet theory (commensurable)

Quantum master equation

$$\frac{d}{dt}\rho = L(t)\rho \quad \text{with} \quad L(t) = L_0 + L_1 \cos(n\Omega t) + L_1' \cos(n'\Omega t)$$

Two-color Floquet theory (commensurable)

Quantum master equation

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- long-time solution periodic, “Floquet solution with eigenvalue 0”

$$\rho(t) = \rho(t + 2\pi/\Omega) = \sum_k e^{-ik\Omega t}\rho_k$$

→ homogeneous set of equations for ρ_k

→ ρ_0, time-averaged expectation values

Two-color Floquet theory (incommensurable)

- $\frac{d}{dt}\rho = L(t)\rho$ with

$$L(t) = L_0 + L_1 \cos(\Omega t) + L_1' \cos(\omega t)$$

- auxiliary angular coordinate $\omega t \longrightarrow \theta$

$$\frac{d}{dt}\mathcal{P} = \mathcal{L}(t,\theta)\mathcal{P}$$

$$\mathcal{L}(t,\theta) = L_0 + L_1 \cos(\Omega t) + L_1' \cos(\theta) - \omega \frac{\partial}{\partial \theta}$$

➔ $2\pi/\Omega$-periodic time-dependence

➔ solve by usual Floquet tools — here: matrix-continued fractions

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➔ $2\pi/\Omega$-periodic time-dependence

➔ solve by usual Floquet tools — here: matrix-continued fractions

- connection $\rho(t) = \mathcal{P}(t,\theta)\big|_{\theta=\omega t}$

cf. t-t' formalism, see Peskin & Moiseyev, J.Chem.Phys. 1993

Summary

- Berry phase and its non-adiabatic generalization
- Floquet theory
  - long-time dynamics: quasienergies $\leftrightarrow$ geometric phase
  - within driving period: Floquet states
- Floquet-Bloch-Redfield theory
  - Floquet $+$ Bloch-Redfield
  - time-independent master equation
  - stationary state
  - susceptibility
- Various
  - readout
  - transport
  - matrix-continued fractions
  - bichromatic drive

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