Optomechanics

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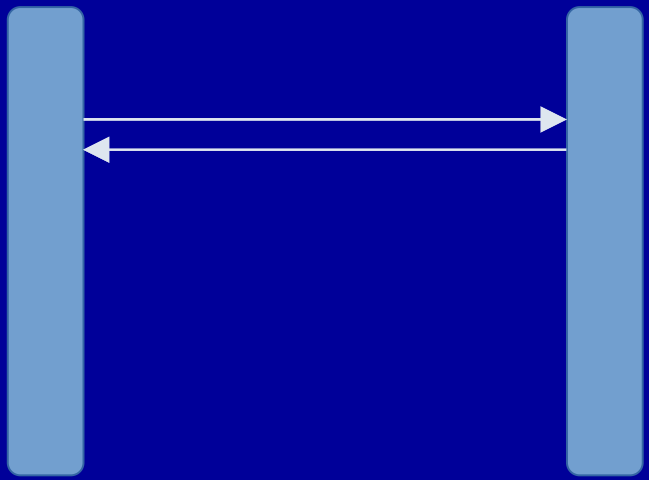
- Optomechanical coupling
- “Classical” optomechanics
- Quantum optomechanics

Optomechanical coupling

Fabry-Perot cavity with one movable mirror:



Movable mirror

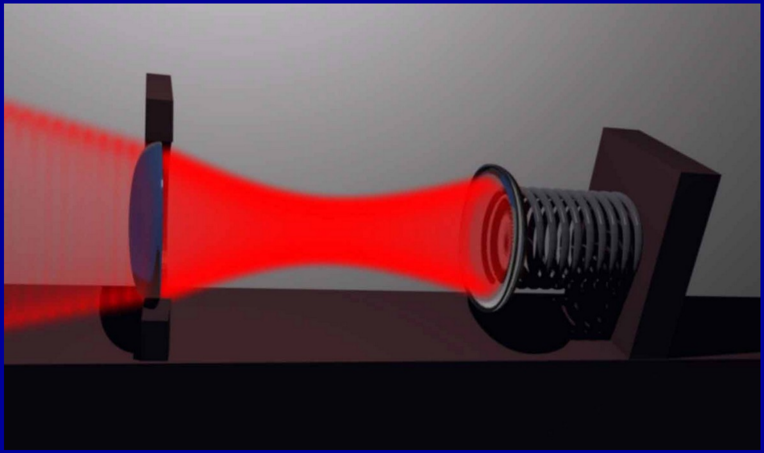
Static mirror

Discrete modes: $\omega = ck = \pi nc/L$: Frequency now depends on the position of the mirror

Scale separation: mechanical motion at MHz and lower; light frequencies at THz (MW at GHz)

$$\omega(x) \to \omega + \frac{\partial \omega}{\partial x} x$$

Radiation pressure



$$\hat{H} = \hbar\left(\omega_c + \frac{\partial \omega_c}{\partial x} x\right)\hat{a}^\dagger \hat{a} + \frac{M\omega^2 x^2}{2}$$

$$\hat{x} = x_{ZPM}\left(\hat{b} + \hat{b}^\dagger\right)$$

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} - \hbar g_0 \hat{a}^\dagger \hat{a}\left(\hat{b}^\dagger + \hat{b}\right)$$

Cavity

Mechanical resonator

Radiation pressure coupling

Coupling

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} - \hbar g_0 \hat{a}^\dagger \hat{a} \left(\hat{b}^\dagger + \hat{b}\right)$$

Sideband-resolved regime

$$\kappa_c, \kappa_m \ll \omega_m \ll \omega_c$$

Where is g_0?

Weak coupling ← → Strong coupling

g_0 - single-photon coupling

Caveats

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} - \hbar g_0 \hat{a}^\dagger \hat{a} \left(\hat{b}^\dagger + \hat{b}\right)$$

- “Membrane in the middle” configurarion: Interaction is also quadratic in *x*
- Surface acoustic waves in MV cavities – can in principle be made resonant

Linearization

$$\hat{H}_{int} = -\hbar g_0 \hat{a}^\dagger \hat{a} \left(\hat{b}^\dagger + \hat{b} \right)$$

Driving and linearization: $\langle \hat{a} \rangle = \sqrt{n_{cav}}$

If $n_{cav} \gg 1$ we can linearize the interaction

$$\hat{a} = \sqrt{n_{cav}} + \delta \hat{a}$$

$$\hat{H}_{int} = -\hbar g \left(\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger \right) \quad g = g_0 \sqrt{n_{cav}}$$

g - multi-photon coupling

Rotating frame approximation: fails at ultrastrong coupling $g \sim \omega_m$

Coupling

$$H_{\text{int}} = -\hbar g_0 \hat{a}^\dagger \hat{a}(b^\dagger + b) \to -\hbar g(\hat{a}^\dagger + \hat{a})(b^\dagger + b)$$

Non-resonant? Depends how we drive. $g = g_0\sqrt{n_{cav}}$

In the rotating frame: $\sqrt{n_{cav}} \propto e^{i\omega_d t}; a \propto e^{i\omega_{cav} t}; b \propto e^{i\omega_m t}$

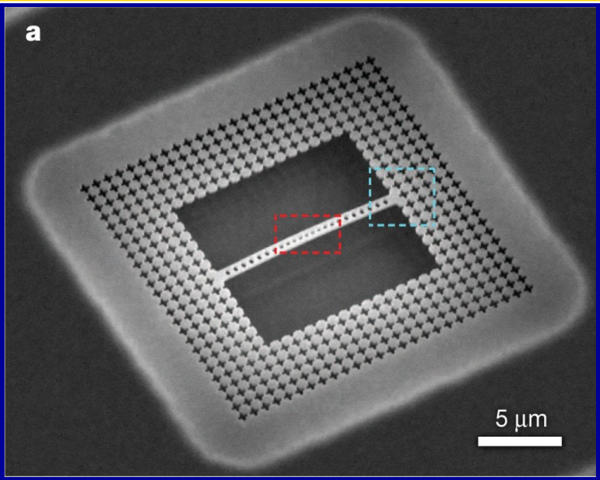
Red-detuned drive: $\omega_d = \omega_{cav} - \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b + \hat{a} b^\dagger)$$

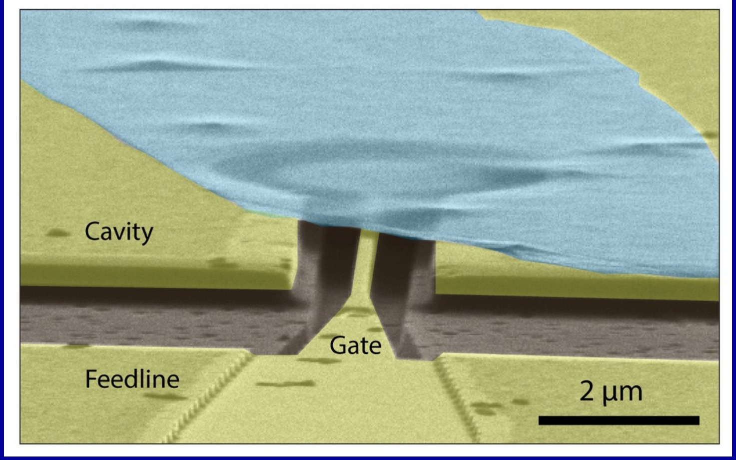
Blue-detuned drive: $\omega_d = \omega_{cav} + \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b^\dagger + \hat{a} b)$$

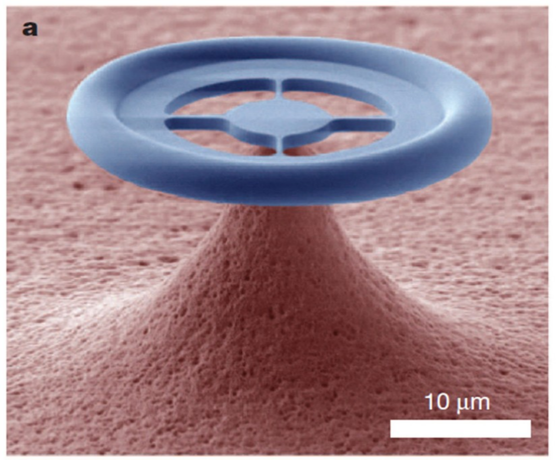
Cavity/circuit optomechanics



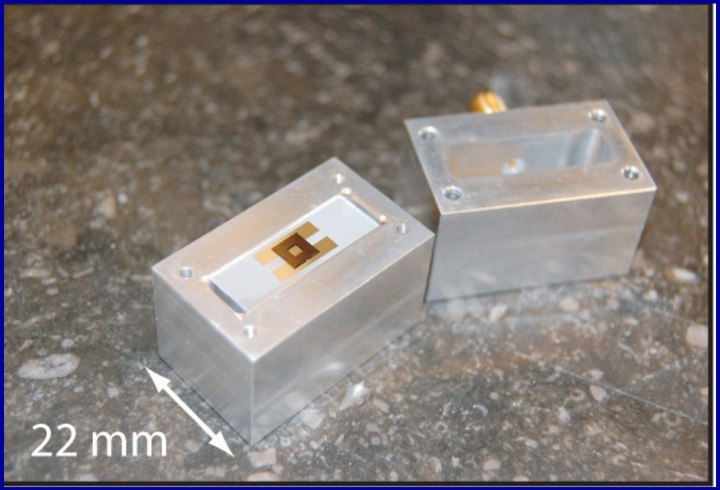
Chan et al, Nature **478**, 89 (2011)



Singh et al, Nature Nanotech. **9**, 820 (2014)



Verhagen et al, Nature **482**, 63 (2012)



Yuan et al, Nature Comms. 6, 8491 (2015)

“Classical” regime

M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86** 1391 (2014)

Static effects

If we only look at the mechanical resonator:
– Equilibrium position is shifted
– Frequency is renormalized
– Damping coefficient is renormalized
– Non-linearity appears and can lead to instabilities

Same with the cavity: frequency shift and renormalization of the damping

For example, the shift of the mechanical frequency is (low drive):

$$\delta\omega_m = 8\Delta \left(\frac{g_0}{\kappa_c}\right)^2 \frac{n_{cav}}{[1+(2\Delta/\kappa_c)^2]^2} \left[1 + (2(g_0 x/x_{ZPM} + \Delta)/\kappa_c)^2\right]$$

$\Delta = \omega_d - \omega_c$ - detuning

Optomechanical cooling

Driving at the red sideband: both creates and absorbs vibration quanta (phonons)

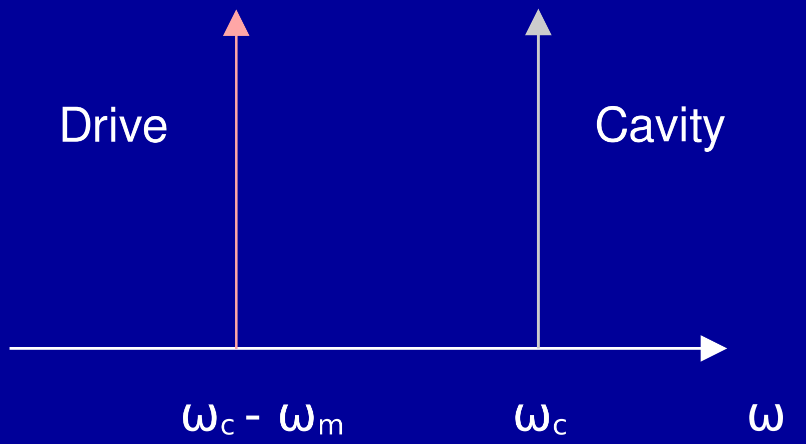
Created: Absorbed by the cavity

Without thermal environment:

$$\bar{n}_{\min} = \left(\frac{(\kappa_c/2)^2 + (\Delta - \omega_m)^2}{(\kappa_c/2)^2 + (\Delta + \omega_m)^2} - 1 \right)^{-1}$$

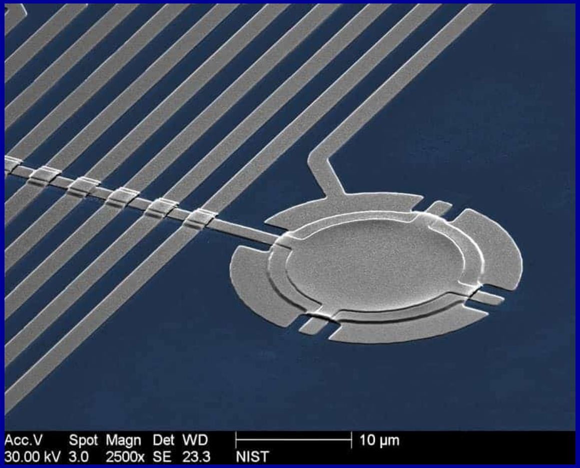
Sideband-resolved regime:

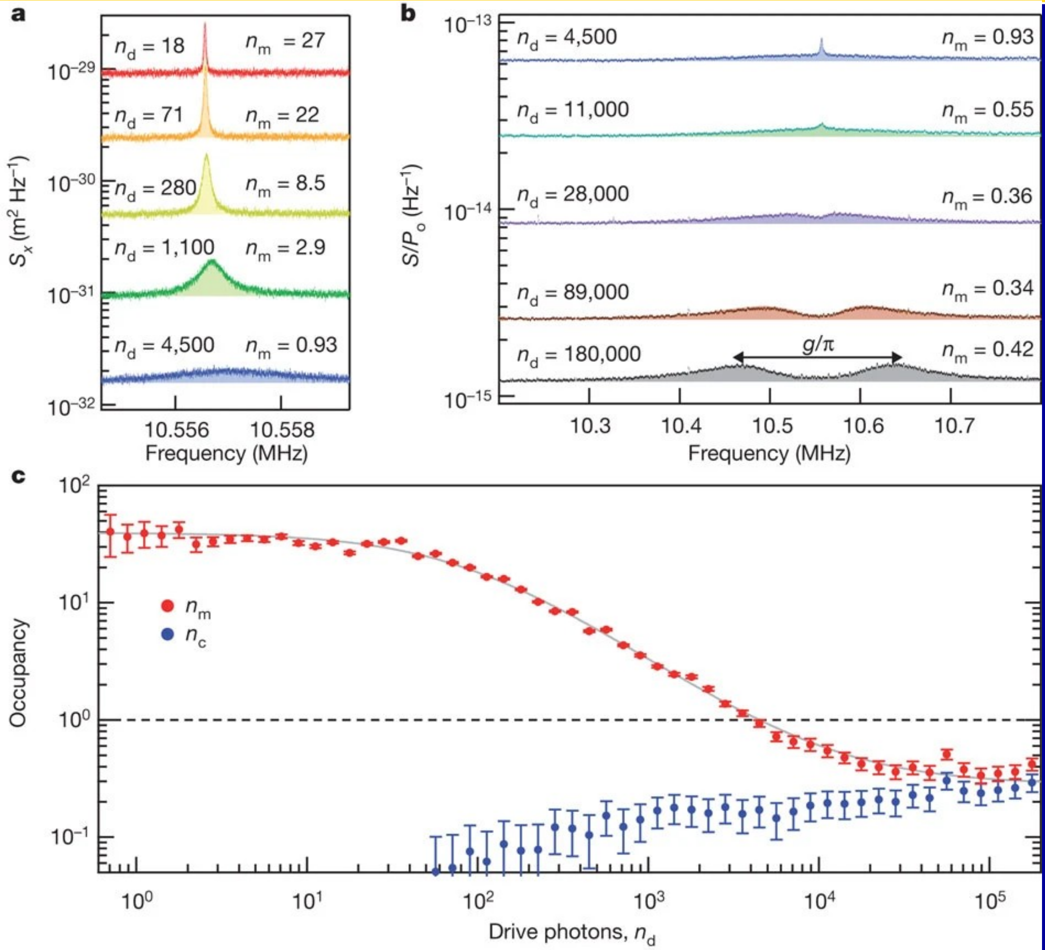
$$\kappa_c \ll \omega_m \rightarrow \bar{n}_{\min} = \left(\frac{\kappa_c}{4\omega_m} \right)^2 \ll 1$$



F. J. Marquardt, P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. **99**, 93902 (2007).

Optomechanical cooling





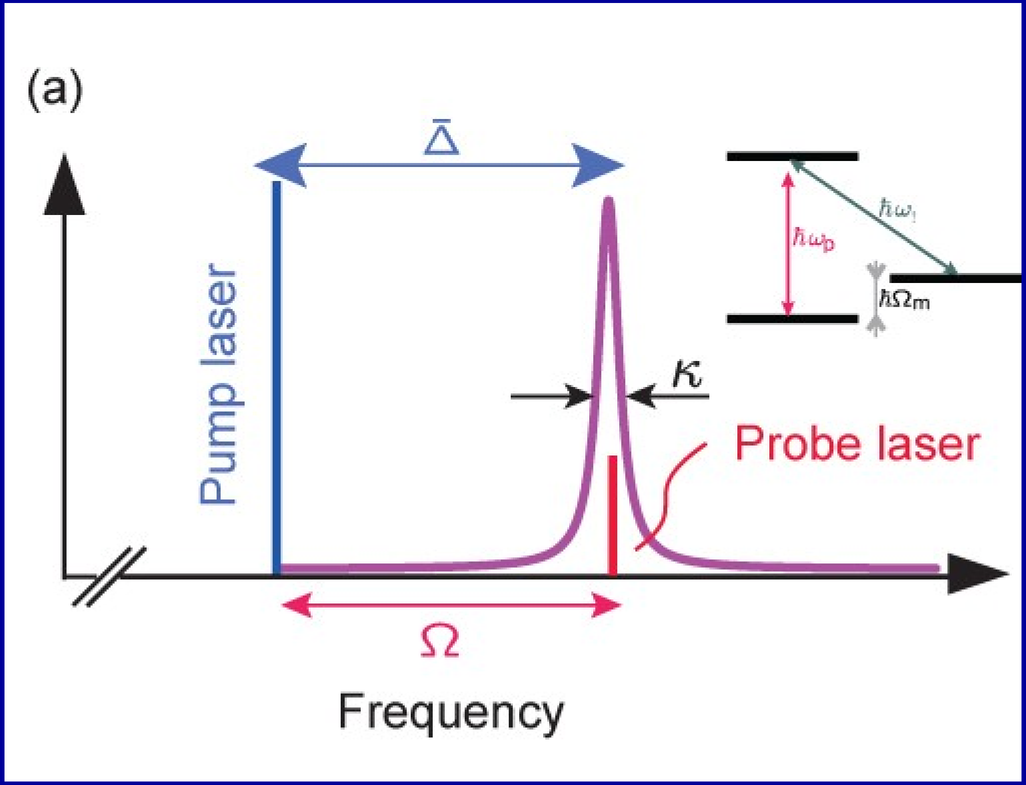
MW cavity; occupation is extracted from the position noise spectral density

Cavity: $f_c \sim 7.5\,\mathrm{GHz}$

Mechanical resonator: $f \sim 10\,\mathrm{MHz}$

J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature **475**, 359 (2011)

Optomechanically induced transparency



Strong red-detuned drive
Probe laser measures the transmission around the cavity resonance

S. Weis, R. Rivière, S. Deléglise, E. Gavartin, O. Arcizet, A. Schliesser, and T. J. Kippenberg, Science **330**, 1520 (2010)

Optomechanically induced transparency

Langevin equations for the creation/annihilation operators in the frame rotating with the drive:

$$\frac{d\hat{a}}{dt} = \left(i\Delta - \frac{\kappa}{2} \right) \hat{a} - iG\hat{x}\hat{a} + \sqrt{\kappa_{ext}}\, s_{in} + \sqrt{(1-\eta_c)\kappa}\,\delta\hat{s}_{vac}(t)$$

Detuning and dissipation in the cavity

Input signal

Quantum noise

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}$$

$$\eta_c = \frac{\kappa_{ext}}{\kappa}$$

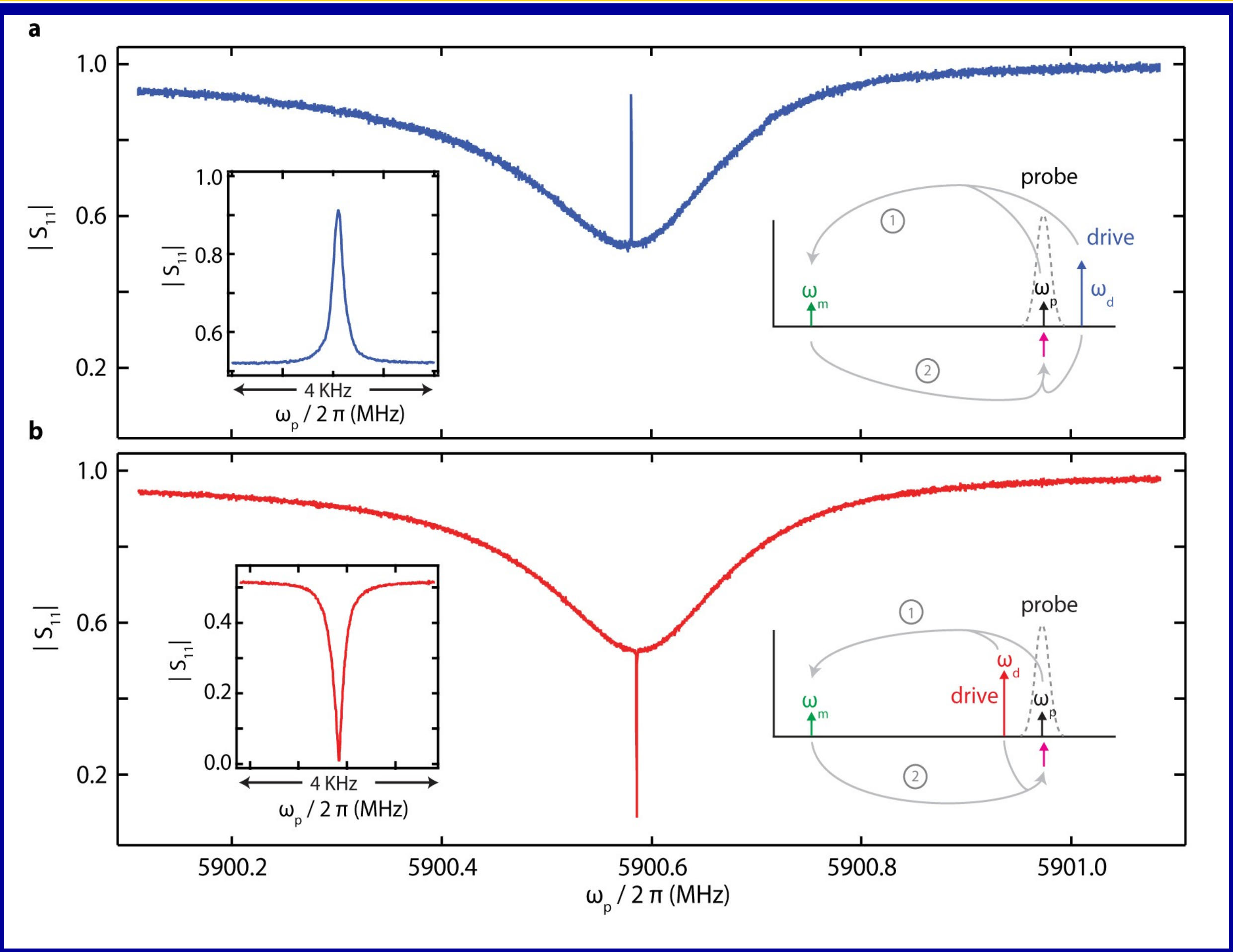
$$\frac{d\hat{p}}{dt} = -m\omega_m^{\;2}\hat{p} - \hbar G\hat{a}^{\dagger}\hat{a} - \Gamma_m \hat{p} + \delta F_{th}(t)$$

Coupling

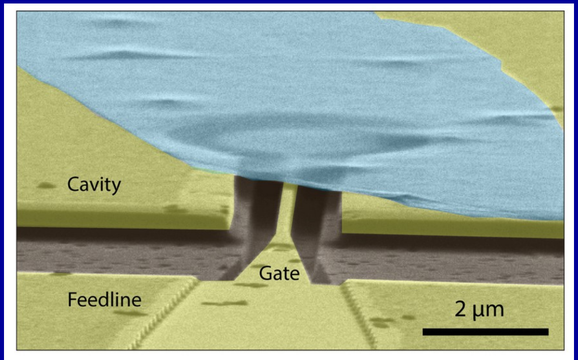
Mechanical dissipation

Thermal noise

Optomechanically induced (transparency) reflection



Constructive interference between the two probes results in OMIT



V. Singh, S. J. Bosman, B. H. Schneider, YMB, A. Castellanos-Gomez, and G. A. Steele, Nature Nanotechnology **9**, 820 (2014)

Quantum optomechanics

Y. Chu and S. Gröblacher,Appl. Phys. Lett. **117**, 150503 (2020)

Quantum detection of mechanical oscillations

Can we see quantum effects in mechanical motion?

Issues:

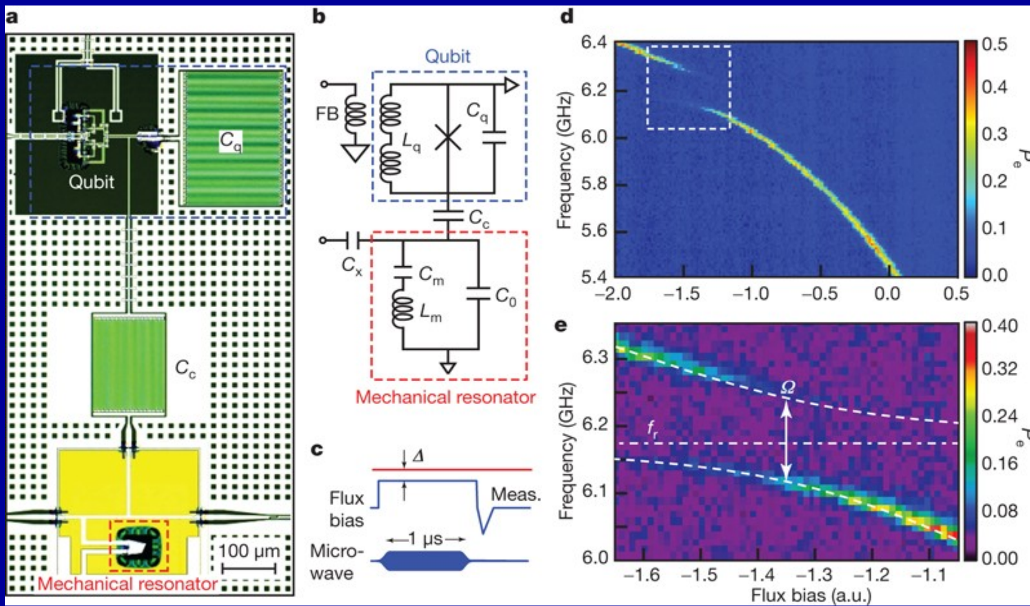
1. Need low temperatures $k_B T \ll \hbar\omega$

$$T = 1K \quad \Rightarrow \quad \omega \gg 100\,\text{GHz}$$

Either need to cool the mechanical resonator down or need to work with very high frequencies

2. Need to decide what are the signatures of the quantum behavior and need a quantum detector to measure them

Coupling to a qubit



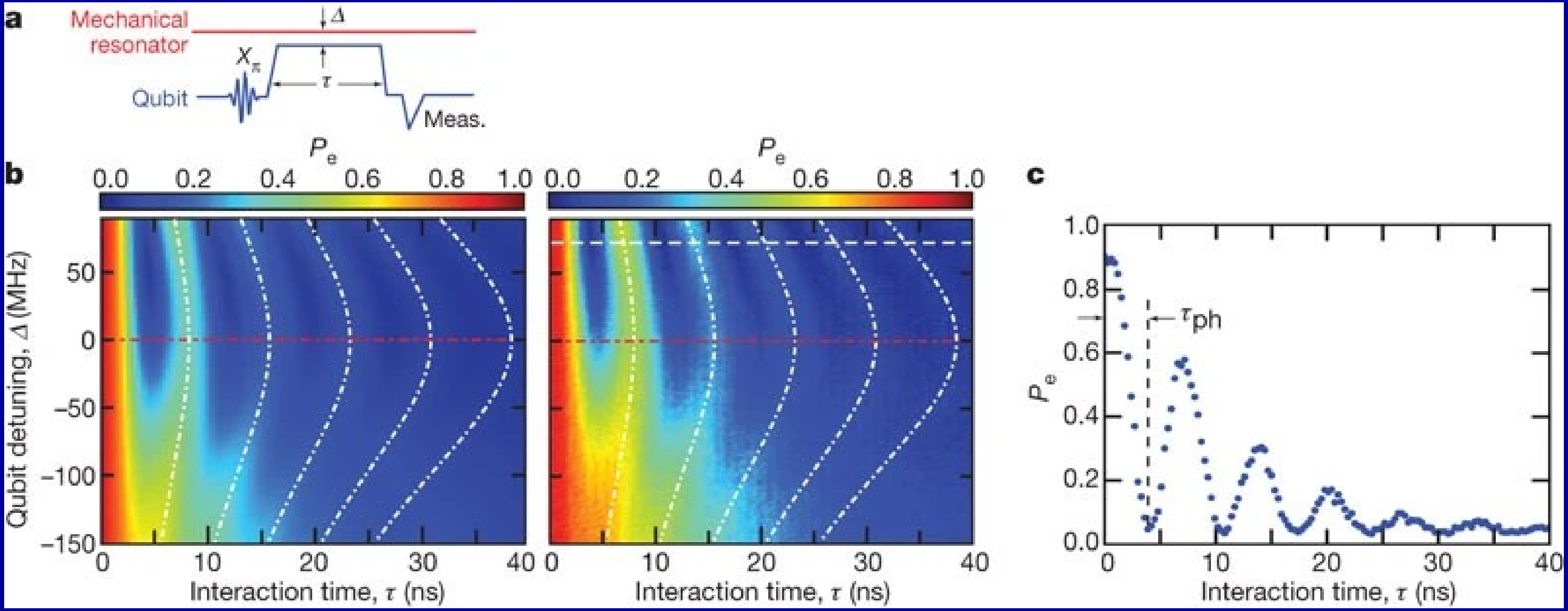
A mechanical resonator capacitively coupled to a superconducting qubit $f \sim 6\,\text{GHz}$

Coupling mechanism:
Mechanical motion modifies the capacitance of the qubit
g ~ 380 MHz

Decay:
mechanical resonator ~ 25MHz
qubit ~ 60 MHz

A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, A. N. Cleland, Nature **464**, 697 (2010)

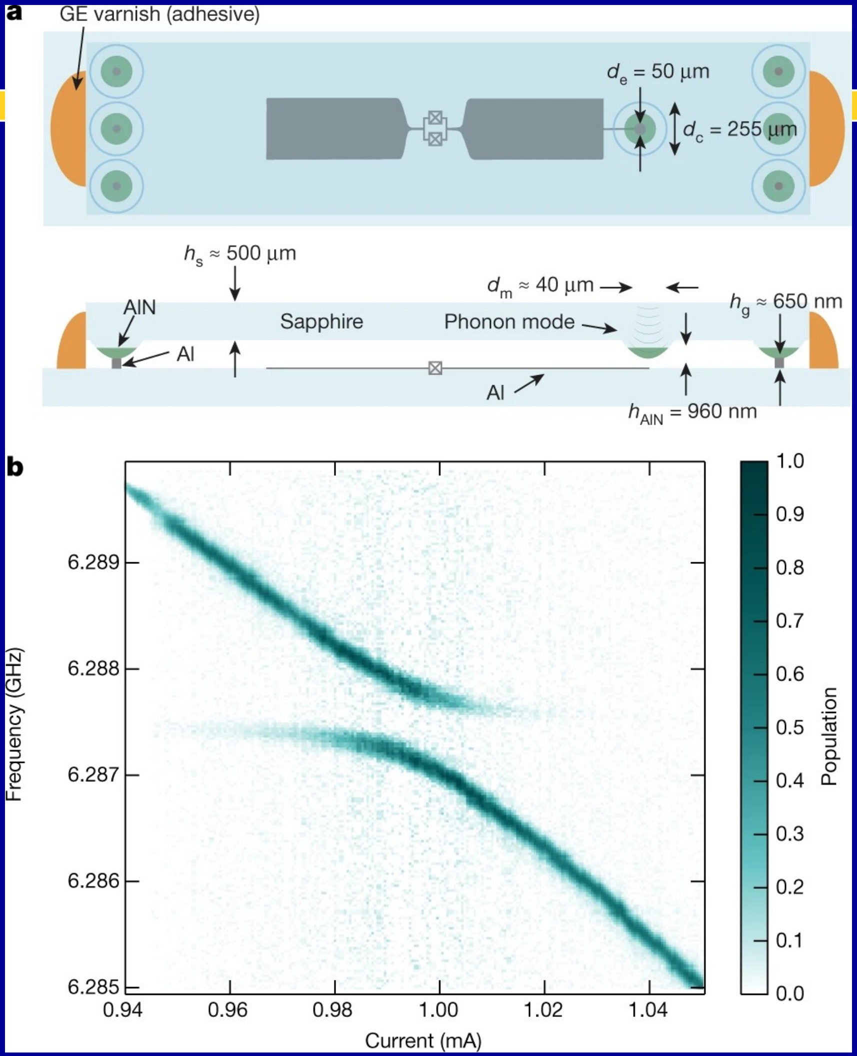
Quantum state swap



c: Probability of the qubit to be in the excited state

A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, A. N. Cleland, Nature **464**, 697 (2010)

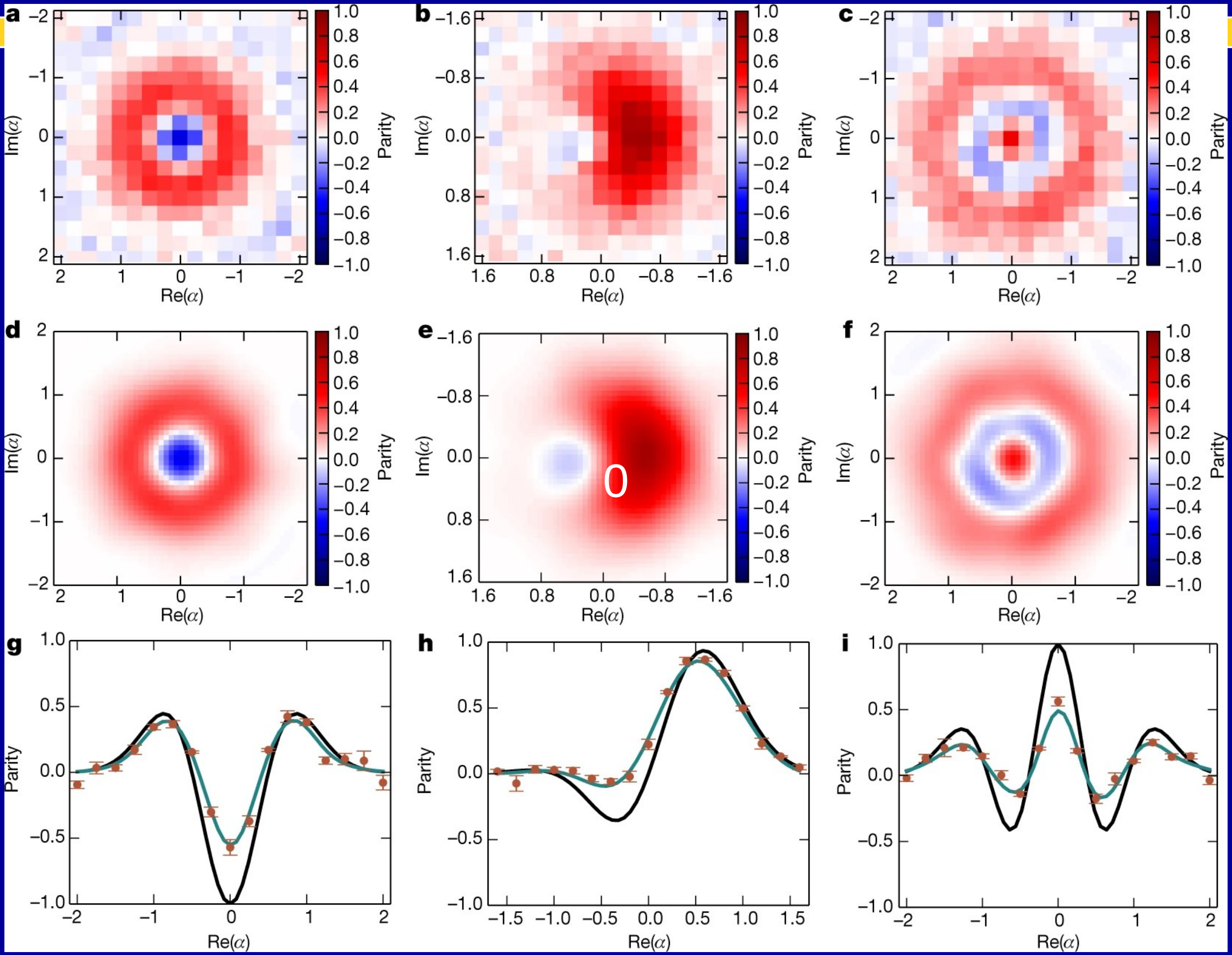
Quantum acoustodynamics



Coupling: $g \sim 2\pi \times 350$ kHz

Y. Chu, P. Kharel, T. Yoon, L. Frunzio, P. T. Rakich, and R. J. Schoelkopf, Nature **563**, 666 (2018)

Wigner tomography

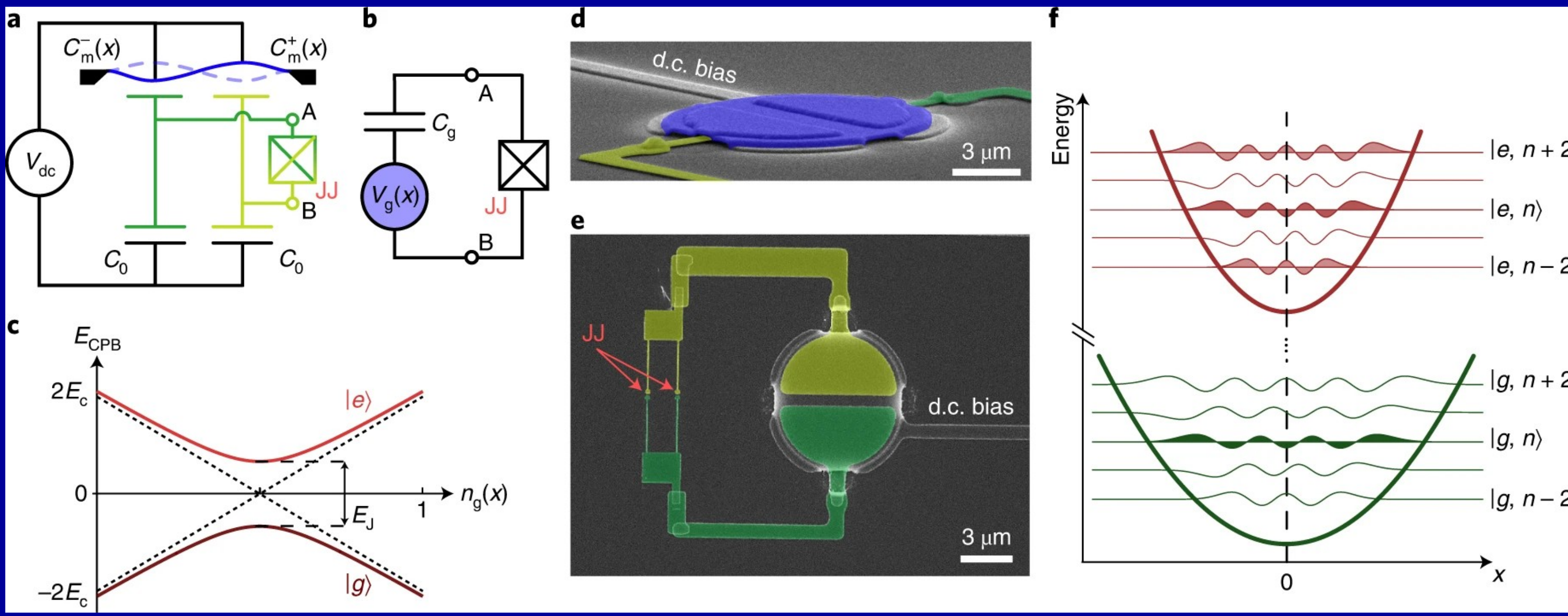


Y. Chu, P. Kharel, T. Yoon, L. Frunzio, P. T. Rakich, and R. J. Schoelkopf, Nature **563**, 666 (2018)

Where do we go from here?

- Non-linear interactions and non-trivial quantum states
- Stronger coupling
- Other platforms

Non-linear coupling



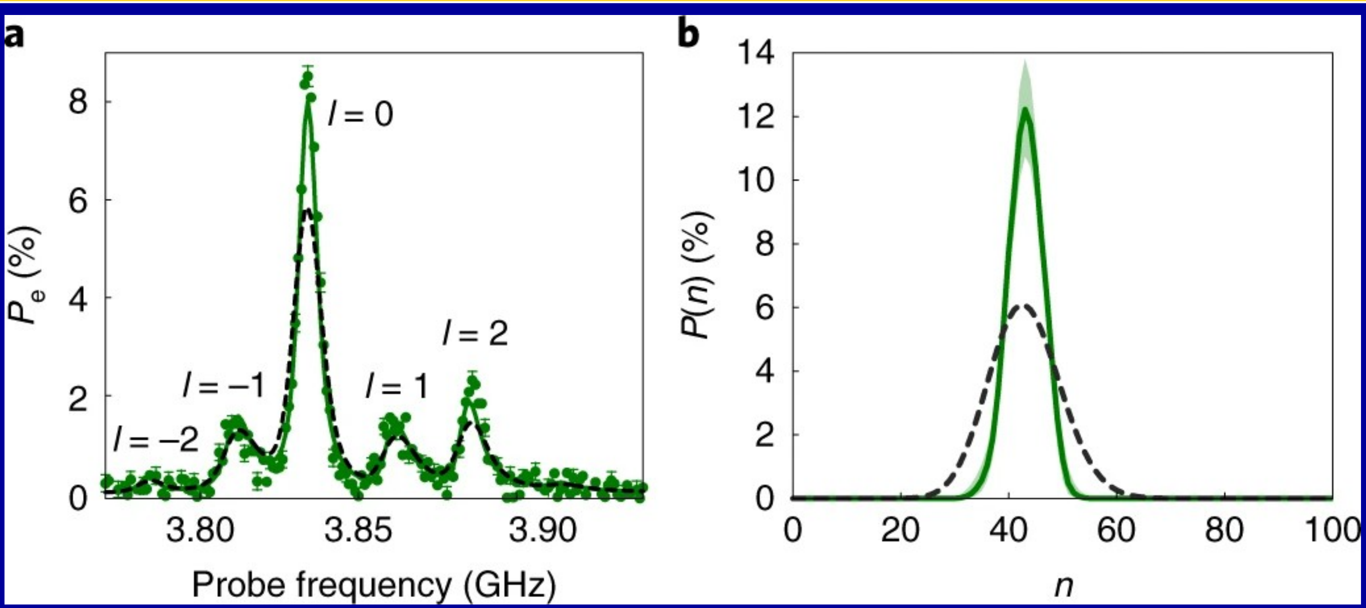
Qubit: ~2π x 3.8 GHz
Resonator: ~2π x 25 MHz
Quadratic coupling: ~2π x 520 kHz

Working at a point when the charging energy has a minimum

$$\hat{H}_{int} \propto \hat{\sigma}_z x^2 \propto \hat{c}^\dagger \hat{c} \hat{b}^\dagger \hat{b}$$

X. Ma, J. J. Viennot, S. Kotler, J. D. Teufel, and K. W. Lehnert, Nature Physics **17**,322 (2021)

Sub-poissonian phonons

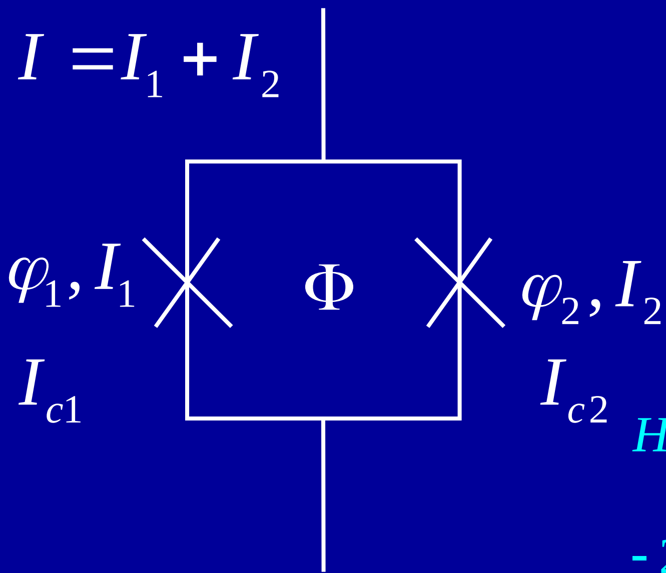


"Classical" physics: Bosons super-Poissonian, fermions sub-Poissonian

Sub-Poissonian bosons: quantum signature

X. Ma, J. J. Viennot, S. Kotler, J. D. Teufel, and K. W. Lehnert, Nature Physics **17**,322 (2021)

SQUID as a microwave cavity



Coupling: Area is modified by the mechanical motion
Not possible to quantize in the general form!

$$H = \frac{M\dot{x}^2}{2} + \frac{M\omega^2 x^2}{2} + \frac{C\Phi_0^2\dot{\varphi}^2}{8\pi^2}$$
$$- 2E_J\sqrt{\cos^2(\varphi\_) + \alpha^2\sin^2(\varphi\_)}\cos\left(\varphi - \arctan\left(\alpha\tan\varphi\_\right)\right)$$

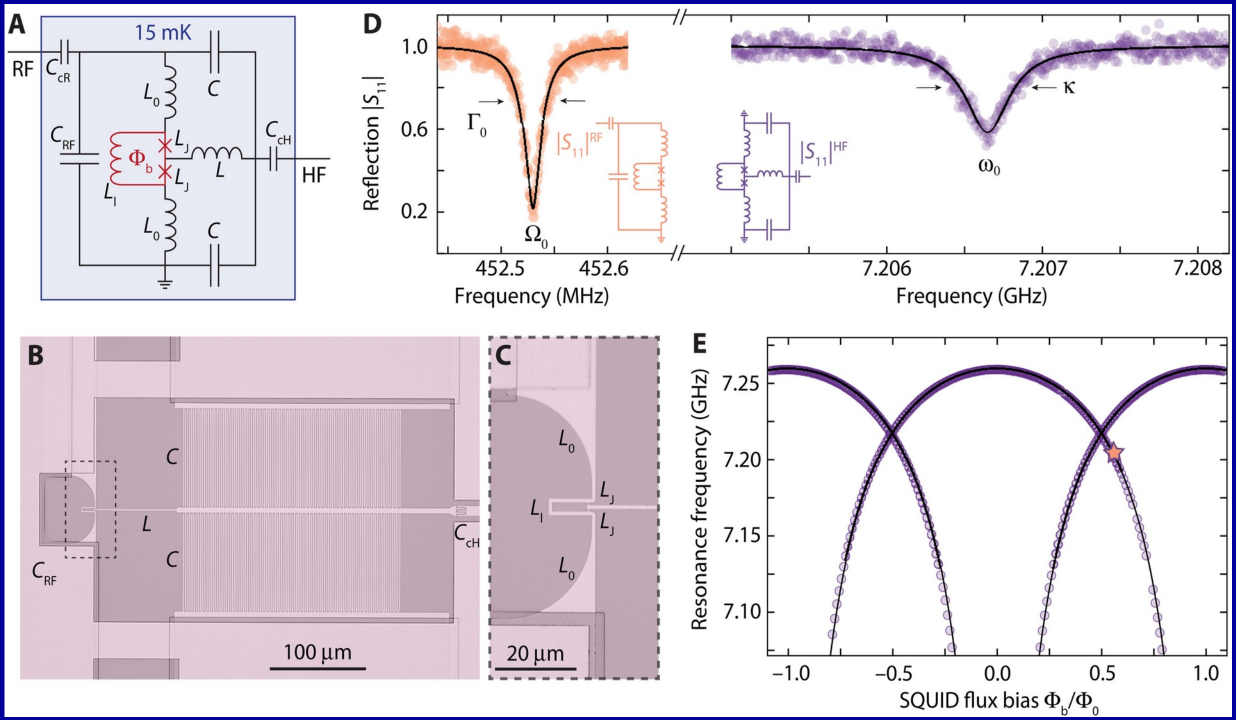
$\varphi\_ = \pi\Phi/\Phi_0 + \xi x$ – reduced flux modified by the displacement

If the SQUID frequency is much higher than the mechanical frequency

$$\hat{H}_{int} = g_Q \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} + g_{RP} \hat{a}^\dagger \hat{a} \left(\hat{b}^\dagger + \hat{b}\right)$$ - tunable

O. Shevchuk, G. A. Steele, and YMB, Phys. Rev. B **96**, 014508 (2017)

Inductive coupling



Qubit: ~2π x 7.2 GHz

Resonator: ~2π x 450 MHz

Single-photon coupling: ~2π x 160 kHz

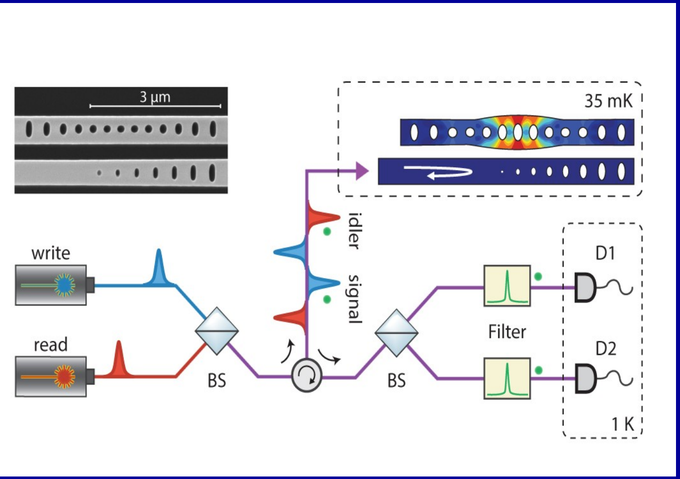
Cavity linewidth: ~2π x 350 kHz

I. Corveira Rodrigues, D. Bothner, and G. A. Steele, Sci. Adv.7, sciadv.abg6653 (2021)

Quantum interferometry

Two-point correlation function:

$$g^{(2)}(\tau) = \frac{\left\langle b^\dagger(t) b^\dagger(t+\tau) b(t) b(t+\tau) \right\rangle}{\left\langle b^\dagger(t) b(t) \right\rangle^2}$$



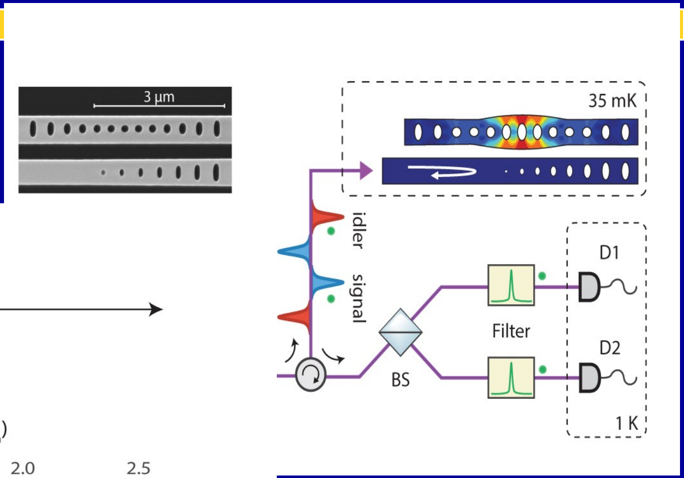
Signature of non-classical states: $g^{(2)}(0) < 1$

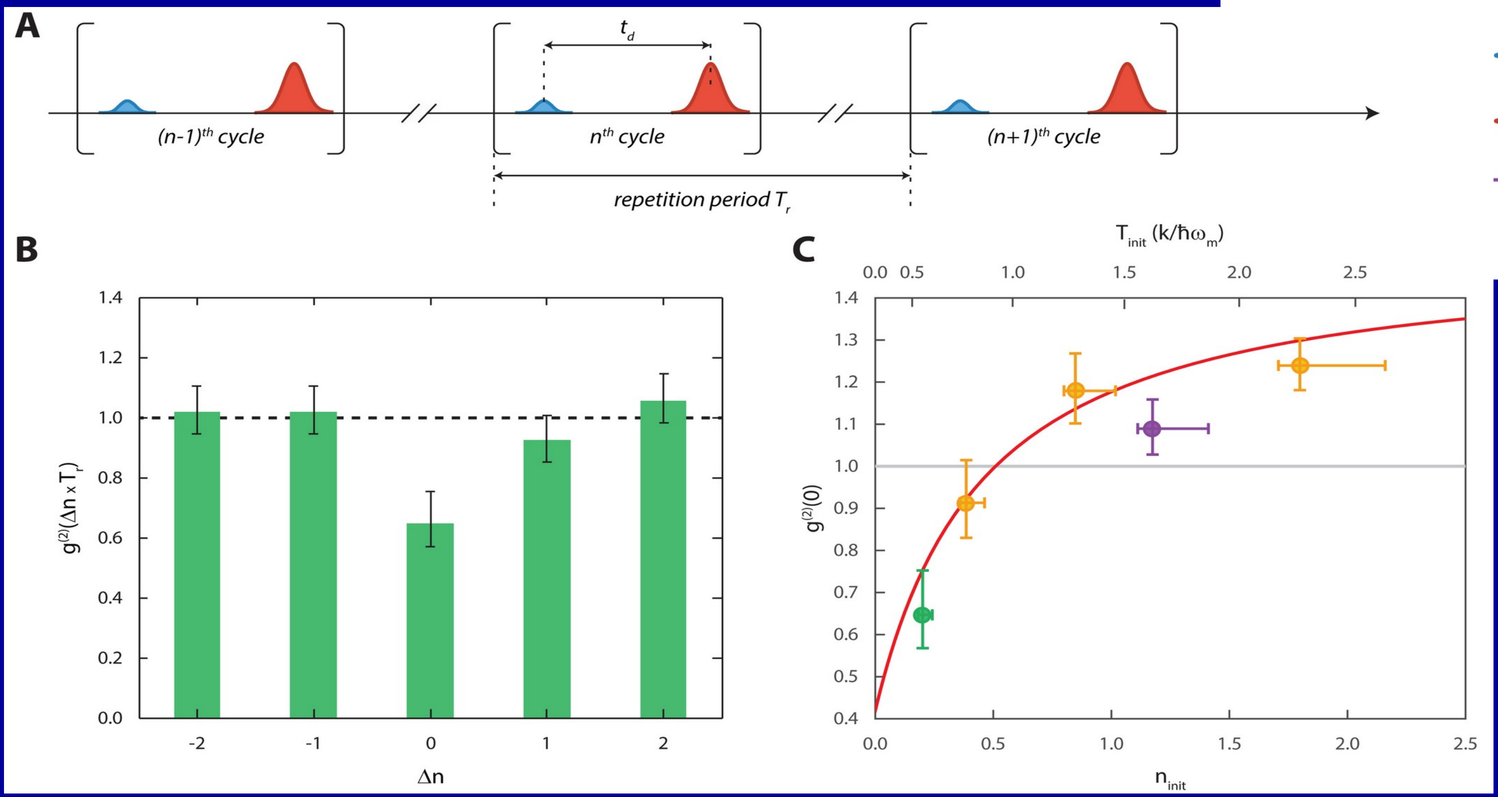
Generally: $0 < g^{(2)}(0) < 2$

S. Hong, R. Riedinger, I. Marinkovic, A. Wallucks, S. G. Hofer, R. A. Norte, M. Aspelmeyer, S. Gröblacher, Science **358**, 203 (2017)

Hanbury Broun – Twiss interferometry

Signature of non-classical states: $g^{(2)}(0) < 1$





S. Hong, R. Riedinger, I. Marinkovic, A. Wallucks, S. G. Hofer, R. A. Norte, M. Aspelmeyer, S. Gröblacher, Science **358**, 203 (2017)

Conclusions

- Mechanical resonatiors interact with electromagnetic radiation:
  - Via radiation pressure
  - Can be strong coupling
  - This can be used for phenomena such as cooling or OMIT
- Quantum properties of magnons: interaction with a qubit or quantum optics with photons