Closing the Quantum Metrology Triangle: Bloch oscillations in Josephson junctions

Fabian Hassler

RWTH Aachen University, Germany

Quantum metrology triangle

time: time/frequency (f)

Cesium-133 atom: 9,192,631,770 Hz
precision 10^{-15}

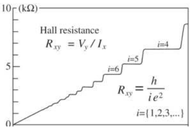
electric: Voltage (V) Current (I)

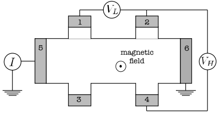
Connection between V and I

electric: Voltage (V) | Current (I)

$$\rho_{xy} = \frac{V_L}{I} = \frac{R_K}{\nu}$$

von Klitzing constant: $R_K = 25812.8074593045\,\Omega \quad (= h/e^2)$



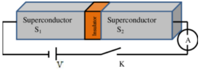


Connection between f and V

time/frequency (f)

Voltage (V)

Josephson junction



electrical symbol

Josephson relations:

$$I = I_c \sin(\phi)$$

$$V = \frac{\hbar}{2e}\dot{\phi}$$

connects freq. and voltage

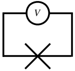
dissipationless element

pot. energy stored

$$\Delta E = -\int VI\,dt = -\frac{\hbar}{2e}\int I_c \sin(\phi) d\phi = U(\phi_e) - U(\phi_a)$$

$$U(\phi) = -\frac{\hbar I_c}{2e}\cos(\phi) = -E_J\cos(\phi)$$

AC Josephson effect



Josephson relations:

$$I = I_c \sin(\phi)$$

$$V = \frac{\hbar}{2e}\dot{\phi}$$

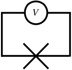
$$\phi = \frac{2eV}{\hbar}t$$

$$I = I_c \sin\left(\frac{2eV}{\hbar}t\right)$$

AC-current source

1GHz ↔ 2μeV

AC Josephson effect



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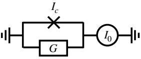
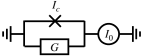
realistic circuit



$$I_0 = \frac{\hbar G}{2e}\dot{\varphi} + I_c \sin\varphi$$

"realisitic" V(t)

realistic circuit



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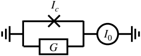


$$I_0 = \frac{\hbar G}{2e}\dot{\varphi} + I_c \sin\varphi$$

$$I_0 = I_c \sin\varphi + \frac{\hbar G}{2e}\dot{\varphi}$$

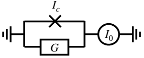
introduce: $j_0 = \frac{I_0}{I_c}$, $\tau = \frac{2e I_c}{\hbar G} t = \omega_G t$

$$j_0 = \sin\varphi + \frac{d\varphi}{d\tau}$$

$$\tau = \int^{\varphi(\tau)} \frac{d\varphi'}{j_0 - \sin\varphi'} = \frac{2}{\sqrt{j_0^2 - 1}} \arctan\left[\frac{\sqrt{j_0^2-1}}{j_0 + 1} \tan\left(\frac{\varphi}{2} + \frac{\pi}{4}\right)\right]$$

"realisitic" V(t)

realistic circuit



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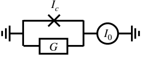
period: $T = \int_0^{2\pi} \frac{d\varphi}{j_0 - \sin\varphi} = \frac{2\pi}{\sqrt{j_0^2-1}}$

voltage: $V(t) = \frac{\hbar}{2e}\dot{\varphi} = \frac{I_c}{G}\frac{d\varphi}{d\tau} = \frac{I_c}{G}\frac{j_0^2 - 1}{j_0 - \cos(2\pi\tau/T)}$

$$f_J = \frac{2eI_c}{\hbar G}\frac{1}{T} = \frac{2eI_c}{\hbar G}\sqrt{j_0^2 - 1} = \frac{2eV_{\rm dc}}{\hbar}$$

"realisitic" V(t)

realistic circuit



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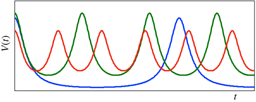
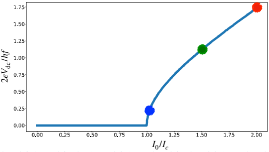
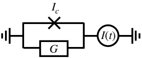




AC-locking/synchronization

realistic circuit



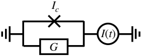
$$I(t) = \frac{\hbar G}{2e}\dot{\varphi} + I_c \sin\varphi$$

$$I(t) = I_0 + I_1 \sin(2\pi f t)$$

synchronizing the AC Josephson effect with frequency $f_J = 2eV/h$ to the external drive

AC-locking/synchronization

realistic circuit



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synchronizing the AC Josephson effect with frequency $f_J = 2eV/h$ to the external drive

voltage standard (2019):

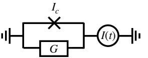
$$V = K_J f$$

Josephson constant

$$K_J = 483597.84841698 \text{ GHz/V} \quad \left(= \frac{2e}{h}\right)$$

AC-locking/synchronization

realistic circuit



time/frequency (f)

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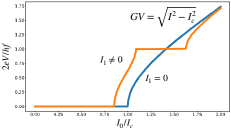
precision 10^{-8}

synchronizing the AC Josephson effect with frequency $f_J = 2eV/h$ to the external drive

Voltage (V)



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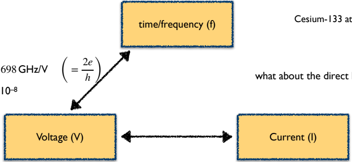
SI-system

time:

Cesium-133 atom: 9,192,631,770 Hz
precision 10^{-15}



$K_J = 483597.84841698$ GHz/V $\left(= \dfrac{2e}{h} \right)$

precision 10^{-8}

what about the direct link (linking I to f)

electric:

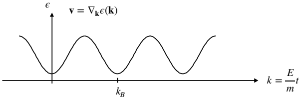
$R_K = 25812.8074593045\,\Omega \quad (= h/e^2)$

precision 10^{-8}

Bloch oscillations

semiclassical equations of particle in band-structure:

$$m\dot{\mathbf{k}} = \mathbf{E}$$
$$\mathbf{v} = \nabla_{\mathbf{k}}\epsilon(\mathbf{k})$$



similar to Josephson relations:

$$\frac{\hbar}{2e}\dot{\phi} = V$$
$$I = I_c \sin(\phi)$$

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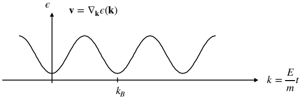

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current I and velocity v oscillates with a period:

$$T_B = \frac{E}{k_B m}$$

that corresponds to the frequency: $f_B = \frac{k_B m}{E}$

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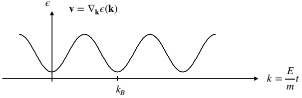

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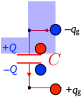
synchron. connects frequency f and f_B

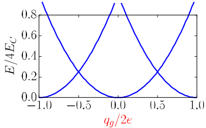
measurement of $k_B m/E$

to connect f and I, we need to find a system where f_B is connected to charge

Cooper pair box

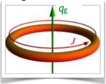


$$H = \frac{(Q + q_g)^2}{2C}$$

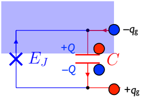


the polarization charge acts as a vector potential

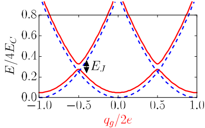
mechanical analog



Cooper pair box



$$H = \frac{(Q + q_g)^2}{2C} - E_J \cos(h\Phi/2e)$$



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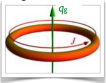
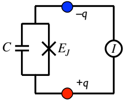


Bloch oscillations

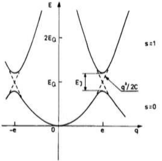
Phase slip regime

- Small Josephson junction with a shunt capacitance



$E_C = e^2/2C \lesssim E_J$

$E_Q = E_C$



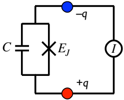
- Strong separation of bands
- Lowest energy band depends strongly on q
- Approximately given by $E(q) \approx -E_Q \cos(\pi q/e)$

K. K. Likharev and A. B. Zorin 1985

Bloch oscillations

Phase slip regime

- Small Josephson junction with a shunt capacitance



$$E_C = e^2/2C \lesssim E_J$$

$$E(q) \approx -E_Q \cos(\pi q/e)$$

voltage: $V(q) = \dfrac{dE(q)}{dq} = V_c \sin(\pi q/e)$ $\quad V_c = \dfrac{\pi E_Q}{e}$

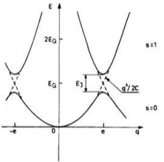
$$\dot{q} = I$$

$$V = V_c \sin(\pi q/e)$$

voltage oscillates with the Bloch frequency

$$f_B = \frac{I}{2e}$$

$$E_Q = E_C$$

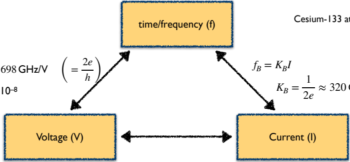


K. K. Likharev and A. B. Zorin 1985

Closing the triangle

time:

Cesium-133 atom: 9,192,631,770 Hz
precision 10–15



$K_J = 483597.84841698$ GHz/V $\left(= \frac{2e}{h} \right)$

precision 10–8

$f_B = K_B I$

$K_B = \frac{1}{2e} \approx 320$ GHz/pA

electric:

$R_K = 25812.8074593045\,\Omega \quad (= h/e^2)$

precision 10–8

K. K. Likharev and A. B. Zorin 1985

Early experiment

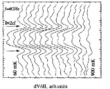
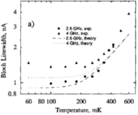


Fig. 4. Temperature dependence of dV/dI versus I for RF power 8 dB at $f = 4$ GHz. The curves are shifted in horizontal axis. The arrows show the expected positions of the Bloch peaks at $I = \pm 2ef$.



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Physica B 203 (1994) 376–380

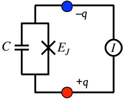
PHYSICA B

Linewidth of Bloch oscillations in small Josephson junctions

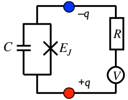
Leonid Kuzmin[a,b,c,*], Yuri Pashkin[a,d], Alexander Zorin[b,c], Tord Claeson[a]

We have carried out a set of measurements of the line width of Bloch oscillations in small Josephson junctions biased through small-size high-ohmic resistors. We have shown that the line width, which we associate with the width of the DC response of the junction to small-signal RF radiation, does not decrease with decreasing temperature below 100–200 mK. This behavior can be explained by a hot electron effect in the resistors. At first glance it seems hard to cool electrons in such a small volume, because a larger geometrical size of resistors

Problem



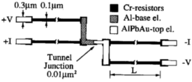
replaced by



heating!!!!!

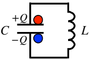
large resistance in oder to localize charge

resistance has to be close to the junction because of parasitic capacitances



Kuzmin, Pashkin, Zorin, Claeson,1994

LC circuit (superconducting)



Hamiltonian $H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$

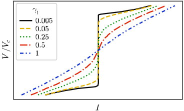
charge Q and flux Φ conjugate $[Q, \Phi] = i\hbar$

Heisenberg relation $\Delta Q \, \Delta \Phi \geq \frac{1}{2}\hbar$

for the ground state, we find $\frac{\Delta Q}{2e} = \frac{Z_0}{\sqrt{2R_K}}$ $\frac{2e\Delta\Phi}{\hbar} = \frac{R_K}{\sqrt{2Z_0}}$ with $Z_0 = \sqrt{L/C}$

- whether charge or phase is a "good" quantum number depends in the ratio of the characteristic impedance $Z_0 = \sqrt{L/C}$ to the von Klitzing constant $R_K = 25\,\mathrm{k\Omega}$
- Charge localization needs large inductances. Demanding since $Z_{\text{vac}} = 377\,\Omega = 2\alpha R_K$

Idea

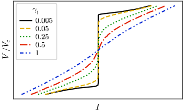


Arndt, Roy, FH, 2018

Idea

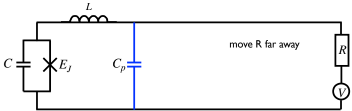
Problem: parasitic capacitances reduce V_c

$$V(q) = \frac{dE(q)}{dq} = V_c \sin(\pi q/e) \qquad V_c = \frac{\pi E_Q}{e} \propto \frac{1}{C + C_p}$$



Arndt, Roy, FH, 2018

Idea



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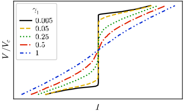
Solution: large impedance L with

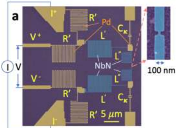
$$Z_0 = \sqrt{L/C_p} > R_K$$

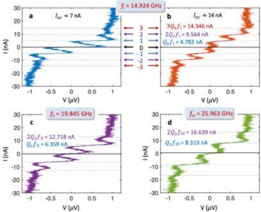
to localize the charges!!!!



Arndt, Roy, FH, 2018

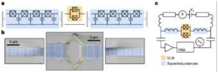
Experiment 1

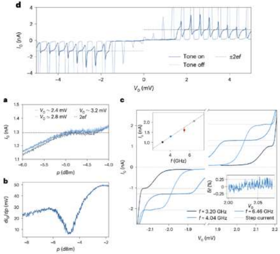




Shaikhaidarov, …, Il'ichev, Astafiev, Nature (2022)

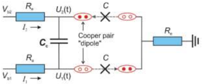
Experiment 2

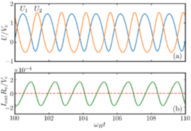


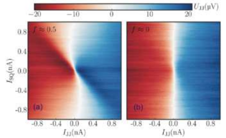


Crescini, …, Roch, Nature Phys. (2023)

Experiment 3



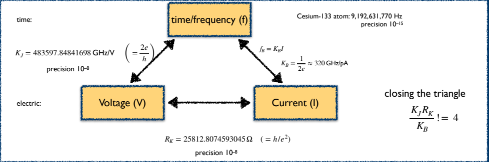




Kaap, Scheer, FH, Lotkhov, (2023) submitted to PRL

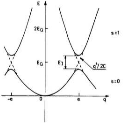
Conclusion

- Bloch oscillations have been observed
- Large (super-)inductances allow to mitigate the problem with temperature
- Accuracy not yet enough to close the quantum metrology triangle



Challenges in Implementation

- Loss of coherence in Bloch oscillations
  - Landau-Zener tunneling into higher bands
  - Thermal noise from large biasing resistor
- Preserving the high impedance environment
  - AC biasing lines introduce parasitic capacitances
  - Mitigated by large on-chip impedance



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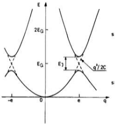
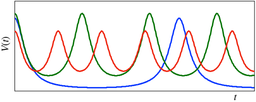


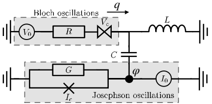
use AC Josephson effect to replace the external drive



On chip AC drive

Using the AC-Josephson effect

- Use Josephson oscillations to drive the phase-slip junction
- Coupling via an LC-resonator



Relevant frequencies	$\omega_R, \omega_G, \omega_0 = (LC)^{-1/2}$
Relaxation timescales	$\tau_L = L/R, \tau_C = C/G$
Resistances	R, G^{-1} $R_c = V_c/I_c, Z_0 = \sqrt{L/C}$

Discussed in context of mutual synchronization of two conjugate quantum variables in:
Hriscu, Nazarov, PRL (2013)

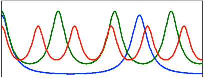
$$\omega_R = \frac{\pi V_c}{eR}, \omega_G = \frac{2eI_c}{\hbar G}$$

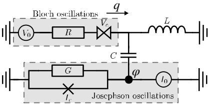
On-chip AC-drive

Using the AC-Josephson effect

- Rigid driving signal for large bias current $I_0 \gg I_c$

  $\Rightarrow \varphi(t) \approx (I_0/I_c)\omega_G t = \omega_J t$
- Effective model in upper loop in case of resonance $\omega_J = \omega_0$





$$\frac{V_0}{V_c} - \frac{Z_0}{R_c}\cos(\omega_0 t) = \frac{1}{\omega_R}\dot{q} + \sin(q) + F(q,t)$$

Back action on the driver

Effective driving strength

On-chip AC-drive

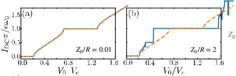
Influence of back action on the Josephson oscillations

- Back action contribution

$$F(q,t) = \frac{\tau_L}{\omega_R}\int \frac{d\omega}{2\pi}\frac{1 - i\omega\tau_C}{1 - i\omega\tau_C - \omega^2/\omega_0^2}(\ddot{q})_\omega e^{-i\omega t}$$

- Negligible in the regime

$Z_0 \ll R$ and $\tau_L \ll \tau_C$



$\omega_0 = 10\sqrt{\omega_R\omega_G}$,

$\omega_R/\omega_G = 100$,

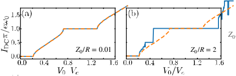
and $\omega_0\tau_C = 10$

On-chip AC-drive

Influence of back action on the Josephson oscillations

- Back action contribution

$$F(q,t) = \frac{\tau_L}{\omega_R}\int \frac{d\omega}{2\pi}\frac{1 - i\omega\tau_C}{1 - i\omega\tau_C - \omega^2/\omega_0^2}\langle \ddot{q}\rangle_\omega e^{-i\omega t}$$



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$Z_0 \ll R$ and $\tau_L \ll \tau_C$

$\omega_0 = 10\sqrt{\omega_R\omega_G}$,

$\omega_R/\omega_G = 100$,

and $\omega_0\tau_C = 10$

- Increases maximum step width ΔV for

$R_Q = 6.45\,\mathrm{k}\Omega \ll \mathrm{R} \lesssim \mathrm{Z}_0$

On-chip AC-drive

Influence of back action on the Josephson oscillations

- Back action contribution

$$F(q,t) = \frac{\tau_L}{\omega_R} \int \frac{d\omega}{2\pi} \frac{1 - i\omega\tau_C}{1 - i\omega\tau_C - \omega^2/\omega_0^2} (\ddot{q})_\omega e^{-i\omega t}$$

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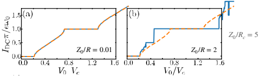
- Increases maximum step width ΔV for

$R_Q = 6.45\,\mathrm{k\Omega} \ll \mathrm{R} \lesssim \mathrm{Z}_0$

Not achievable at $Z_0 \lesssim 10\,\mathrm{k\Omega}$



$\omega_0 = 10\sqrt{\omega_R \omega_G}$,

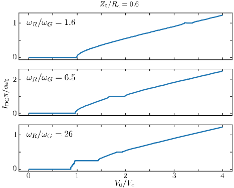
$\omega_R/\omega_G = 100$,

and $\omega_0 \tau_C = 10$

On chip AC-drive

Influence of back action on the Josephson oscillations

- For increasing ω_R/ω_G
  - Steps move closer to the blockade region
    - $\Rightarrow$ Smaller bias voltage V_0
  - Increased sensitivity to the effective driving strength Z_0/R_c



$$\omega_0 = 4\sqrt{\omega_R \omega_G},$$

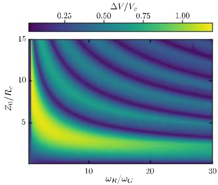
$\omega_0 \tau_L = 4 \cdot 10^{-3}$, and

$\omega_0 \tau_L = 4 \cdot 10^{-2}$

On chip AC-drive

Influence of circuit parameters

- Parameter regions with robust dual Shapiro steps
- At fixed ratio ω_R/ω_G
  - Step width ΔV oscillates for varying Z_0/R_c
  - Faster oscillations for increasing ω_R/ω_G



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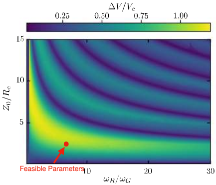

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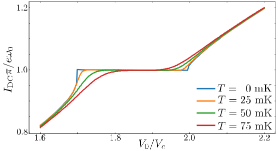
Effects of thermal noise

For realistic circuit parameters

- Steps with Nyqvist-Johnson noise from the resistors



$Z_0/R_c \approx 0.6, \omega_0/\bar{\omega} \approx 4, \omega_0\tau_C \approx 0.04, \omega_0\tau_L \approx 0.004$, and $\omega_R/\omega_G \approx 6.5$

Corresponding to

$L = 0.01\,\mu\text{H}, \text{C} = 100\,\text{fF}, \text{R} = 50\,\text{k}\Omega, \text{G} = 0.1\,\text{S}, \text{I}_\text{c} = 0.1\,\mu\text{A}$, and $\text{V}_\text{c} = 50\,\mu\text{V}$

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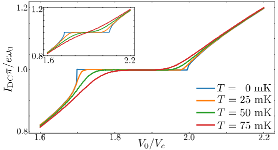

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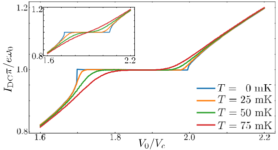
- Increased robustness due to finite back action



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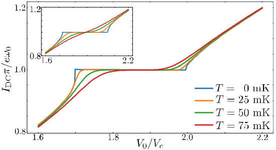
- Feasible circuit parameters yield
  $I_{DC} = 1.6\,\mathrm{nA} \ll 20\,\mathrm{nA}$ at $V_c = 50\,\mu\mathrm{V}$
  ⇒ Negligible Landau-Zener tunnelling



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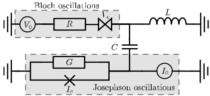
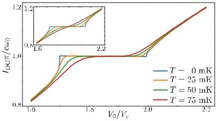
Geigenmüller, Schön, 1988

Conclusion

- Circuit for dual Shapiro steps without an external AC drive
- Effective model equivalent to external AC drive for small back action
- Steps with increased robustness to thermal noise at realistic experimental parameters





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arXiv:2307.03448

Bloch oscillations

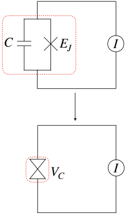
Phase slip regime

- Lowest band can be modeled as a phase-slip junction

$$V = V_c \sin \frac{\pi}{e} Q \overset{\text{Dual}}{\longleftrightarrow} I = I_c \sin \frac{2e}{\hbar} \phi$$

- Critical voltage V_c relates to the width of the lowest band $2E_S$

$$V_c = \frac{\pi}{e} E_S \qquad E_S \approx E_C^{1/4} E_J^{3/4} e^{-\sqrt{8E_J/E_C}}$$

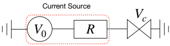


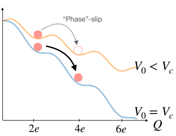
J. Koch et al., Phys. Rev. A 76, 042319 (2007)

Bloch oscillations

In an over-damped phase-slip junction

- High impedance environment
  $$R \gg R_Q = h/4e^2$$
- Localized charge $Q = \langle \hat{Q} \rangle$
- Classical equation of motion
  $$R\dot{Q} = V_0 - V_C \sin \frac{\pi}{e} Q$$
- Relevant scales
  $$\omega_R = \frac{\pi V_c}{eR}, \quad q = \frac{\pi}{e} Q$$

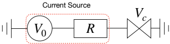


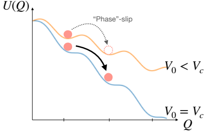


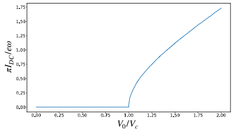
Bloch oscillations

Current to frequency relation

$$I_{DC} = \frac{e}{\pi}\omega_R\sqrt{(V_0/V_c)^2 - 1} = \frac{e}{\pi}\omega_B$$



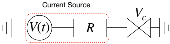


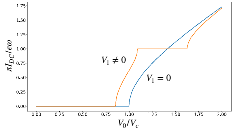


Dual Shapiro steps

Harnessing synchronization for quantum metrology

- Synchronization of Bloch oscillations with an AC-drive
$$V_0 + V_1 \sin \omega t = R\dot{Q} + V_c \sin \frac{\pi}{e} Q$$
- Steps of constant current $I_{DC} = e\omega/\pi$
- Missing link in quantum metrology triangle

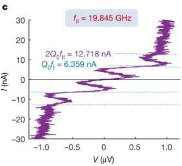




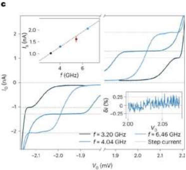
K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985)

Dual Shapiro steps

Experimental demonstrations



R. S. Shaikhaidarov et al, *Nature* **608**, 45–49 (2022)



N. Crescini et al, Nature Phys. **19**, 851 (2023)

Bloch oscillations

Current to frequency relation

- Dimensionless equation of motion

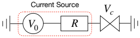
$$\frac{1}{\omega_R}\dot{q} = \frac{V_0}{V_c} - \sin q$$

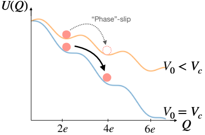
- Time T for a single phase-slip

$$\omega_R T = \int_0^{2\pi} \frac{dq}{V_0/V_c - \sin q} = \frac{2\pi}{\sqrt{(V_0/V_c)^2 - 1}}$$

- DC-current

$$I_{DC} = \frac{2e}{T} = \frac{e}{\pi}\omega_R\sqrt{(V_0/V_c)^2 - 1} = 2ef_B$$

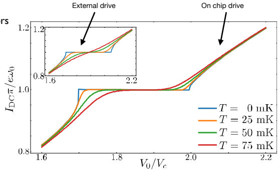




Effects of thermal noise

For realistic circuit parameters

- Steps with Nyqvist-Johnson noise from the resistors
- Increased robustness due to finite back action
- Feasible circuit parameters yield
  $I_{DC} = 1.6\,\mathrm{nA} \ll 20\,\mathrm{nA}$ at $V_c = 50\,\mu\mathrm{V}$
  ⇒ Negligible Landau-Zener tunnelling



U. Geigenmüller and G. Schön, Physica B **152**, 186 (1988)

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Corresponding to

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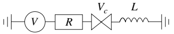
S. Lotkhov and F. Kaap, private communications

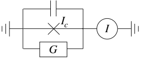
On-chip AC-drive

Using the AC-Josephson effect

- Dual circuit is well established as a voltage standard



Dual circuit

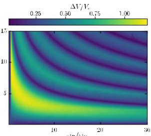


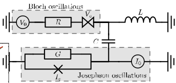
$$V = R\dot{Q} + V_c \sin \frac{\pi}{e} Q + L\ddot{Q}$$

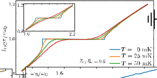
$$\frac{V_0}{V_c} = \frac{1}{\omega_R}\dot{q} + \sin q + \frac{L}{\omega_R R}\ddot{q}$$

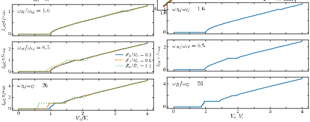
$$I = \frac{\hbar}{2e} G\dot{\varphi} + I_c \sin\varphi + \frac{\hbar}{2e} C\ddot{\varphi}$$

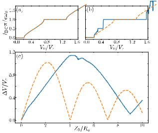
$$j = \frac{1}{\omega_G}\dot{\varphi} + \sin\varphi + \frac{C}{\omega_G G}\ddot{\varphi} \quad \text{with } \omega_G = \frac{2eI_c}{\hbar G}$$


$\Delta V/V_c$
0.25
0.50
0.75
1.00
Z_0/R_c
ω_R/ω_G


Bloch oscillations
V_0
R
V_c
L
C
G
I_0
I_c
Josephson oscillations


$T = 0$ mK
$T = 25$ mK
$T = 50$ mK
$Z_0/R_c = 0.6$


$\omega_R/\omega_G = 1.6$
$\omega_R/\omega_G = 6.5$
$\omega_R/\omega_G = 26$
$Z_0/R_c = 0.3$
$Z_0/R_c = 0.6$
$Z_0/R_c = 1.2$
V_0/V_c


(a)
(b)
(c)
V_0/V_c
$\Delta V/V_c$
Z_0/R_c